



# THE GURUKUL INSTITUTE

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## ELASTIC AND SIMPLE HARMONIC MOTION

### Longitudinal And Shearing Stress

1. Which is more elastic (i) Steel or Rubber, (ii) Water or air?
2. A steel wire of length 4m and diameter 5mm is stretched by 5kg- wt find the increase in its length, if the Young's modulus of steel wire is  $2.4 \times 10^{12}$  dyne/cm<sup>2</sup>.
3. A cable is replaced by another one of same length and material but twice the diameter. How will this affect the elongation under a given load? How does this affect the maximum load it can support without exceeding the elastic limit?
4. Develop an approximate relationship between the change in pressure and the density of a liquid.
5. The bulk modulus of water is  $2.3 \times 10^9$  N/m<sup>2</sup>
  - (a) Find its compressibility.
  - (b) How much pressure (in atmosphere) is needed to compress a sample of water by 0.1%?

### Elastic Potential Energy In a Stretched Wire

6. A long steel wire, when suspended from a rigid end and attached with a heavy block of mass M at the other, stretches by a length L. Work done by gravity is MgL whereas elastic potential energy stored in the stretched wire is  $(1/2)mgL$ . Is this a violation of law of conservation of energy.
7. A steel wire of 4.0m length is stretched through 2.0mm. The cross sectional area of the wire is  $2.0 \text{ mm}^2$ . If Young's modulus of steel is  $2.0 \times 10^{11}$  N/m<sup>2</sup>, find:
  - a) the energy density of wire
  - b) the elastic potential energy stored in the wire.
8. A thin, uniform rod of length L is rotated with constant angular speed  $\omega$  about an axis passing through its one end and perpendicular to the length of rod.  
Find the increase in the length.

[ Density of material is  $\rho$  and Young's modulus is Y.]

9. A dumb-bell ( connecting rod – uniform) having mass  $(2M+2M+M)$  is hung from a vertical support (Two Cu wires and a Fe wire have equal length same area of cross section.) so that the system is symmetrically arranged.  
Calculate tension in all wires. [Given that  $Y_{Cu} : Y_{Fe} = 1:2$ ]

### Simple Harmonic Motion And Harmonic Oscillator

10. (i) A particle oscillates between A and B through O as shown. Will its motion be simple harmonic?  
(ii) A highly super elastic ball bouncing on a hard floor has motion that is approximately periodic. In what way is it similar to SHM and in what way is it different?
11. Any real spring has some mass, how will the frequency of oscillation of a spring- block system change. Discuss qualitatively.
12. A spring is cut into two equal parts. What will be the difference in time period of the spring pendulum thus formed with any one of the cut part and from the original spring pendulum?

### Projection of Uniform Circular Motion on a Diameter

13. A small particle is kept on a fixed smooth concave glass surface of radius of curvature R placed in the vertical plane. Show that if the particle is displaced from the equilibrium position and released, it executes simple harmonic motion. Also, find its period ( displacement being very small compared to R).

### Characteristics Of SHM

14. Determine whether or not the following quantities can be in the same direction for a simple harmonic motion.
  - a) Displacement & Velocity
  - (b) Velocity & acceleration
  - (c) Displacement & acceleration
15. A particle executes simple harmonic motion about the point  $x=0$ . At time  $t=0$ , it has displacement  $x=2$  cm and zero velocity. If the frequency of motion is  $0.25 \text{ sec}^{-1}$ , find (a) The time period, (b) angular frequency, (c) the amplitude, (d) maximum speed, (e) the displacement at  $t=3$  sec and (f) the velocity at  $t=3$  sec.
15. If an SHM is represented by the equation  $x=10 \sin[\pi t + \pi/6]$  in SI units, determine its amplitude, time period and maximum velocity  $v_{\max}$ .
16. A particle executes SHM with a time period of 4 s. Find the time taken by the particle to go directly from its mean position to half of its amplitude.
17. Two particles move parallel to x- axis about the origin with the same amplitude and frequency. At a certain instant, they are found at distance  $(A/3)$  from the origin on opposite sides but their velocities are found to be in the same direction. What is the phase difference between the two?

### Energy of a Body in SHM

18. What is the frequency of the K.E. of a body in SHM if the time period of SHM is  $T$ ?

19. A particle of mass  $4\text{ gm}$  lies in a potential field given by,  $V = 200x^2 + 150$  ergs/gm.

Deduce the frequency of its vibration.

20. A particle executes SHM

(a) What fraction of total energy is kinetic and what fraction is potential when displacement is one half of the amplitude?

(b) At what value of displacement are the kinetic and potential energies equal?

### SIMPLE PENDULUM

21. At what point in the motion of a simple pendulum is the string tension greatest? At what point is it least? What in case of a conical pendulum?

22. How will the frequency of small oscillation of a simple pendulum change:

(i) a point of suspension moves horizontally in the plane of oscillation with acceleration  $a$ .

(ii) If it moves vertically upward with an acceleration ' $a$ '.

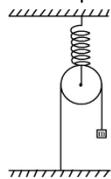
(iii) If it moves vertically down with acceleration  $a < g$ .

22. Show that the period of a oscillation of a simple pendulum at depth  $h$  below earth's surface is inversely proportional to  $\sqrt{(R - h)}$ , where  $R$  is the radius of earth. Find out the time period of a second pendulum at a depth  $R/2$  from the earth's surface?

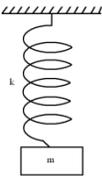
23. Out of two clocks, one based on simple pendulum and the other on spring pendulum. Which one will show correct time on the moon as written on earth?

### Combination of Springs

24. In the given diagram pulley is massless and frictionless. Strings and spring are ideal. If string is taut during the motion of the block of mass ' $m$ '. Calculate the time period of small oscillation.



25. A spring mass system is shown in the figure. The spring stretches  $2\text{ cm}$  from its free length when a force of  $10\text{ N}$  is applied. This spring is stretched  $10\text{ cm}$  from its free length, when a body of a mass  $m = 2\text{ kg}$  is attached to it and released from rest at time  $t = 0$ . Find : (a) the force constant of the spring, (b) the time period and frequency of vibration, (c) the amplitude of vibration, (d) the initial velocity and acceleration, (e) the maximum velocity and acceleration, (f) the spring force at the two extreme positions of the body.



### Physical Pendulum

26. What is the equivalent length of a simple pendulum for the physical pendulum given above?

27. A uniform square lamina of side  $2a$  is hung up by one corner and oscillates in its own plane which is vertical. Find the length of the equivalent simple pendulum.

28. A disc is suspended at a point  $R/2$  above its centre. Find its period of oscillation.

### PROBLEMS

1. A light rod of length  $200\text{ cm}$  is suspended from the ceiling horizontally by means of two vertical wires of equal lengths tied to its ends. One of the wire is made of steel and is of cross sectional area  $0.1\text{ cm}^2$  and the other is of brass of cross sectional area  $0.2\text{ cm}^2$ . Find out the position along the rod at which weights may be hung to produce

(a) equal stress in both wires, (b) equal strains in both wires.

$$(Y_{\text{steel}} = 20 \times 10^{11} \text{ dynes/cm}^2 \text{ and } Y_{\text{brass}} = 10 \times 10^{11} \text{ dynes/cm}^2)$$

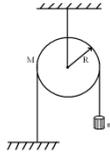
2. A heavy, uniform metallic bar of mass  $M$  is being supported by three rods as shown in the figure. Area of cross section of all the three rods are same and equal to  $A$ . The lengths and young moduli of the rods are indicated. Find the stress developed in each of the three rods.

3. A horizontal platform vibrates up and down with a simple harmonic motion of amplitude  $20\text{ cm}$ . At what frequency will an object kept on the platform just lose contact with the platform? [Take  $g = 9.81\text{ m/s}^2$ .]

4. A trolley of mass  $M$  oscillates with some time period about its mean position when it is connected with an ideal spring of stiffness constant  $K$ . If an unknown mass  $m$  is gently placed at the top of trolley, its time period is found to be  $T$  [between the two blocks friction is sufficient to maintain small oscillation], find unknown mass.

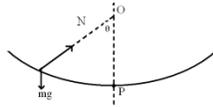
Also, calculate the coefficient of static friction so that the upper mass does not slip even for a sizeable amplitude A.

5. In the arrangement shown in the figure, a heavy pulley of mass M is suspended from a spring of stiffness constant K and the mass m is suspended from a string tightly wound on the pulley. If the mass m is given a small downward displacement and released, find the time period of small oscillations. (Take pulley as disc) and string does not slip on the pulley.



6. Determine the period of oscillation of mercury of mass  $m = 200 \text{ g}$  poured into a bent tube whose right arm forms an angle  $\theta = 30^\circ$  with the vertical. The cross-sectional area of the tube  $S = 0.50 \text{ cm}^2$ . The viscosity of mercury is to be neglected.

7. A hollow sphere of radius  $r$  rolls without slipping in a hemisphere of radius  $4r$ . Calculate the frequency of small oscillations, about point P as shown in figure.



8. A horizontal spring – block system of mass M executes simple harmonic motion. When the block is passing through its equilibrium position, an object of mass m is put on it and the two move together. Find the new amplitude and frequency of vibration.

9. A solid cylinder attached to a horizontal massless spring can roll without slipping along a horizontal surface. Show that if the cylinder is displaced and released, it executes SHM find the corresponding time period.

#### OBJECTIVE

1. Which of the following functions does not represent SHM?

- a)  $\sin^2 \omega t - \frac{1}{2} \cos^2 \omega t$       b)  $\sin^2 \omega t$       c)  $\sin \omega t + 2 \cos \omega t$       d)  $\sin \omega t + \cos 2\omega t$

2. The potential energy  $U(x)$  of a particle executing SHM is given by

- a)  $U(x) = k(x-a)^2 / 2$       b)  $U(x) = k_1 x + k_2 x^2 + k_3 x^3$       c)  $U(x) = A e^{-bx}$       d)  $U(x) = \text{constant}$

3. The kinetic energy of a particle executing SHM will be equal to  $(1/8)^{\text{th}}$  of its potential energy when its displacement from the mean position is

- a)  $A\sqrt{2}$       b)  $A/2$       c)  $2\sqrt{2}/3 \times A$       d)  $A \times \sqrt{2}/\sqrt{3}$

4. Two SHM's are given by  $y_1 = a \sin[(\pi/2)t + \theta]$  and  $y_2 = b \sin[2\pi t/3 + \theta]$ . The phase difference between these after 1 sec is

- a)  $\pi$       b)  $\pi/2$       c)  $\pi/4$       d)  $\pi/6$

5. A simple harmonic motion has an amplitude A and time period T. The time required by it to travel from  $x=A$  to  $x=A/2$  is

- a)  $T/6$       b)  $T/4$       c)  $T/3$       d)  $T/2$

6. Three masses 700 gm, 500 gm, 400gm are suspended at the end of a spring and are in equilibrium. When 700 gm mass is removed, the system oscillates with a period of 3 seconds. When 500 gm is also removed, it will now oscillates with a period of

- a) 1 sec      b) 2 sec      c) 3 sec      d)  $\sqrt{12}/\sqrt{5}$  sec

7. A simple pendulum with angular frequency  $\omega$  oscillates simple harmonically. The tension in the string at lowest point is T. The total acceleration of the bob at its lowest position is

- a)  $(T/m - g)$       b) zero      c)  $g - T/m$       d)  $T + g/m$

8. The time period of a spring-mass system is T in air. When the mass is partially immersed in water, the time period of oscillation is

- a) T      b)  $< T$       c)  $> T$       d)  $\leq T$

9. A simple pendulum with a solid metal bob has a period T. The metal bob is now immersed in a liquid of density one-tenth that of the bob. The liquid is non-viscous. Now the period of the same pendulum (with its bob remaining in the liquid all the time) will be

- a) T      b)  $(9/10) T$       c)  $\sqrt{10/9} T$       d)  $\sqrt{9/10} T$

10. A block of mass m compresses a spring of stiffness k through a distance  $(L/2)$ . If the block is not connected with the spring and the impact of the block with the vertical wall is elastic, the period of motion of the block is

- a)  $2\pi \sqrt{m/k}$       b)  $(\pi+4) \sqrt{m/k}$       c)  $(1+\pi) \sqrt{m/k}$       d) none of these

11. A simple pendulum has a time period  $T_1$  when observed on the surface of earth and  $T_2$  when taken to a height R above the earth's surface. The ratio  $T_2/T_1$  is

- a) 1      b)  $\sqrt{2}$       c) 4      d) 2

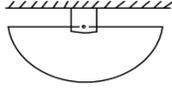
12. One end of a long metallic wire of length L is tied to the ceiling. The other end is tied to a massless spring of spring constant k. A mass m hangs freely from the free end of the spring. The area of cross section and the Young's modulus of

the wire are A and Y, respectively. If the mass is slightly pulled down and released, it will oscillate with a time period T equal to:

- a)  $2\pi\sqrt{(m/k)}$       b)  $2\pi\sqrt{m(YA+kL)/(YAk)}$       c)  $2\pi\sqrt{(mYA/kL)}$       d)  $2\pi\sqrt{(mL/YA)}$

13. A uniform semi-circular disc of mass m and radius r is suspended as shown in the figure. If T is the time period of small oscillations about an axis passing through point of suspension and perpendicular to plane of disc. Then T is equal to

- a)  $2\pi\sqrt{3r/8g}$       b)  $2\pi\sqrt{3r\pi/8g}$       c)  $2\pi\sqrt{2r\pi/9g}$       d)  $2\pi\sqrt{2r/9g}$



14. One end of a spring of spring constant k is attached to the centre of a disc of mass m and radius R and the other end of the spring is connected to a rigid wall. A string is wrapped on the disc and the end A of the string is pulled through a distance 'a' and then released. The disc is placed on a horizontal rough surface and there is no slipping at any contact point. What is the amplitude of oscillation of the centre of the disc?

- a) a      b) 2a      c) a/2      d) none of these

15. The displacement y of a particle executing periodic motion is given by  $y = 4\cos^2(t) \sin(1000t)$ . This expression may be considered to be a result of the superposition of

- a) two independent harmonic motions      b) Three independent harmonic motions  
c) four independent harmonic motions      d) five independent harmonic motions

### ASSIGNMENT

#### Section – I

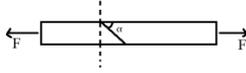
#### PART- A

#### Level – 1

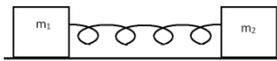
- An elevator is suspended with a cable and its masses is 986.9 kg. If the maximum acceleration of elevator is  $0.2 \text{ ms}^{-2}$  and breaking stress is  $3.14 \times 10^8 \text{ Nm}^{-2}$ , find minimum diameter of the cable.
- Four identical hollow cylindrical columns of steel support a big structure of mass 50,000 kg. The inner and outer radii of each column are 30 cm and 60cm, respectively. Assuming the load distribution to be uniform, calculate the compressional strain of each column. ( $Y_s = 2.0 \times 10^{11} \text{ Pa}$ ,  $g = 10 \text{ m/s}^2$ )
- (a) If density of lead is  $11.4 \text{ g/cm}^3$  and its bulk modulus is equal to  $0.8 \times 10^{10} \text{ N/m}^2$ , calculate change in the density of lead under an excess pressure of  $2 \times 10^4 \text{ N/m}^2$ .  
(b) If stress is increased, to two fold, what will be the ratio of stress and strain.
- (a) A vertical U tube contains water up to a total length 2h. If the water is displaced vertically, find the expression for frequency of the oscillations.  
(b) A particle at the end of a spring executes SHM with a period  $T_1$  while for the another spring the period is  $T_2$ , then what will be the period of oscillation for the motion with the two springs in series.
- (a) A bob of a simple pendulum executes SHM in water with a period T, while the period of oscillation of the bob is  $T_0$  in air. If  $\rho_{\text{bob}} = 4/3 \times 1000 \text{ kg/m}^3$ , then develop a relationship between T and  $T_0$ .  
(b) The time period of a simple pendulum in a stationary lift is  $3\sqrt{3} \text{ s}$ . If lift now accelerates upward with an acceleration of  $(g/2) \text{ ms}^{-2}$ , calculate the new time period of the pendulum.
- A long uniform rope of mass M is suspended from a rigid end, calculate longitudinal strain due to its own wt. Diameter and young's modulus of rope are D and Y respectively. ( $g = 10 \text{ m/s}^2$ )
- A particle is executing SHM along y-axis such that its velocities are  $v_1$  and  $v_2$  at distance  $y_1$  and  $y_2$  from the mean position. Calculate the time period of motion.
- Two SHM's are  $x_1 \text{ (mm, sec)} = 1.0 \sin[\pi/4 (12t+1)]$ , and  $x_2 \text{ (mm, sec)} = 0.5 [\sin 3\pi t + \sqrt{3} \cos 3\pi t]$   
Discuss the ratio of their amplitudes, frequency and find the phase difference.
- Two wires of equal lengths (L) and cross sectional (A) but difference materials of Young's moduli  $Y_1$  and  $Y_2$  respectively, support a load of mass m as shown in the figure. Calculate the time period of small oscillation for which both the strings remains elongated during the complete motion.
- A particle moving with simple harmonic motion has speeds 3 cm/s and 4cm/s at displacements 8cm and 6cm, respectively, from the equilibrium position. Find  
a) the period of oscillation, and  
b) the amplitude of oscillation.

#### LEVEL-II

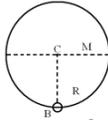
- A steel bar of cross section A is subjected to two equal and opposite pulling forces at the two ends. If there is a plane through the bar making an angle  $\alpha$  as shown, find  
a) the tensile stress      b) the shearing stress at this plane  
c) for what  $\alpha$  the tensile stress is maximum      d) for what  $\alpha$  the shearing strain is maximum



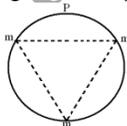
2. A uniform rod of mass  $m$  and length  $L$  is free to rotate about a fixed horizontal axis through its end and perpendicular to its length. Find the period of small oscillation of the rod.
3. A particle of mass ' $m$ ' is located in a uni-dimensional potential field where potential energy of the particle depends on coordinate  $x$  as  $U(x) = U_0(1 - \cos bx)$ ;  $U_0$  and  $b$  are constants. Find the period of small oscillation that the particle performs about equilibrium position.
4. A particle oscillating simple harmonically with an amplitude of 1.5 cm, has a maximum energy of  $0.25 \mu\text{J}$ . At what displacement from the equilibrium position will the particle be acted upon by a force of  $2.5 \times 10^{-5} \text{ N}$ ?
5. A small bob of mass 50g oscillates as a simple pendulum, with an amplitude of 5 cm and period of 2s. Find the velocity of the bob and the tension in the supporting thread, when the velocity of the bob is maximum. [Take  $g = 10 \text{ m/s}^2$ ]
6. A horizontal platform vibrates up and down with a simple harmonic motion of frequency  $(2/\pi) \text{ Hz}$ . Find the maximum permissible amplitude so that an object kept on the platform remains in contact with the platform.
7. Suppose that the spring in the figure has a force constant  $k = 252 \text{ N/m}$ . Let  $m_1 = 1.13 \text{ kg}$  and  $m_2 = 3.24 \text{ kg}$ . Calculate the period of oscillation of the two – body system.



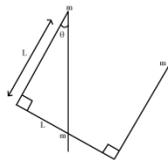
8. A meter stick swinging from one end oscillates with a frequency  $f_0$ . What would be the frequency, in terms of  $f_0$ , if the bottom third of the stick were cut off?
9. Find the distance from the top end of a uniform bar of length 24 cm which is to be mounted on a wall about an axis perpendicular to its length, so that its time period of oscillation will be minimum.
10. A uniform disc of mass  $M = 10 \text{ kg}$  and radius  $R = 0.1 \text{ m}$  is hanging vertically with the help of an axle passing through its centre. A small mass  $m (= 0.10 \text{ kg})$  is stuck at the bottom end B near the rim of the disc. If the disc is now given small angular displacement, find the period of its oscillations. Also find the equivalent length of the simple pendulum.



11. A block of mass ' $m$ ' connected by a spring is kept on a horizontal frictionless floor. When the spring is at its natural length ' $L_0$ ' the block is at a distance ' $a$ ' from the wall. Now, the block is moved by a distance of ' $2a$ ' away from the wall and released. If the collision between the block and wall is elastic, determine the time period of oscillation?
12. A simple pendulum made up of light rod of length  $L$  and bob of mass  $m$  has a spring of force constant  $k$  connected to it at a distance  $x$  below the point of suspension. Find the frequency of vibration of the system.
13. Three particles of the same mass  $m$  are fixed to a uniform circular hoop of mass  $m$  and radius  $R$  at the corners of an equilateral triangle. The hoop is free to rotate in a vertical plane about the point on the circumference opposite to one of the masses. Find the equivalent length of a simple pendulum.

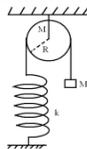


14. An elastic string of natural length  $2a$  can support a certain weight extending to a total length of  $3a$ . One end of the string is now attached to a point on a smooth horizontal table and the same weight is attached to the other end and can move on the table. Show that, if the weight is pulled to any distance and let go, the string slacks after a time  $\pi/2 \sqrt{a/g}$ .
15. A 'U' shaped uniform rod of each of side having length ' $L$ ' and mass ' $m$ ' is hinged from the one end of one of its limbs as shown. Determine:  
 (a) the angle made with vertical when it is in equilibrium position.  
 (b) time period of small oscillations.



16. A spherical ball of mass  $m$  and radius  $r$  rolls without slipping on a concave surface of large radius  $R$ . Find the time period of small oscillations about the lowest point.

17. In the given diagram pulley is disc of mass  $M$  and radius  $R$ , string does not slip on the pulley, calculate the time period of small oscillation of block of identical mass  $M$ .



### Part-B

#### Objective

#### (Multi Choice Single Correct)

- When a mass undergoes SHM, there is always a constant ratio between its displacement and
  - period
  - acceleration
  - mass
  - velocity
- A particle executes SHM with a frequency  $f$ . The frequency with which its K.E. oscillates is
  - $f/2$
  - $f$
  - $2f$
  - $4f$
- Two SHM's are represented by  $y = a \sin(\omega t - kx)$ . The phase difference between the two is
  - $\pi/2$
  - $\pi/4$
  - $\pi/6$
  - $3\pi/4$
- A uniform slender rod of length  $L$ , cross-sectional area  $A$  and Young's modulus  $Y$  is acted upon by the force shown in the figure. The elongation of the rod is
  - $3FL/5AY$
  - $2FL/5AY$
  - $3FL/8AY$
  - $8FL/3AY$
- A simple pendulum with a bob of mass  $m$  is suspended from the roof of a car moving horizontally with constant acceleration ' $a$ '. If bob is at rest in equilibrium position with respect to car then,
  - the string gets inclined with the vertical at an angle  $\tan^{-1}(g/a)$
  - the string makes an angle of  $\tan^{-1}(1 - a/g)$  with the vertical.
  - the tension in the string is  $mg(1 + a/g)$
  - the tension in the string is  $m\sqrt{a^2 + g^2}$
- A particle is executing SHM along the  $x$ -axis given by  $x = A \sin \omega t$ . What is the magnitude of the average acceleration of the particle between  $t=0$  and  $t = (T/4)$  s, where  $T$  is the time period of oscillation.
  - $2\omega^2 A/\pi$
  - $\omega^2 A/\pi$
  - $4\omega^2 A/\pi$
  - none of these
- A thin square plate of mass  $m$  and side ' $a$ ' is hinged about  $AB$  so as to execute small oscillation about  $AB$ . The time period of oscillations is
  - $2\pi \sqrt{a/g}$
  - $2\pi \sqrt{2a/3g}$
  - $2\pi \sqrt{a/6g}$
  - none of these
- A block of mass  $M$  is performing SHM with amplitude  $A$  on a smooth horizontal surface. At the extreme position a small block of mass  $m$  falls vertically and sticks to  $M$ . Then, amplitude of oscillation will be
  - $A$
  - $A \sqrt{M/(M+m)}$
  - $A [M/(M+m)]$
  - $A [(M+m)/M]$
- A body of mass  $m$  is suspended from a rubber cord with force constant  $k$ . The maximum distance over which the body can be pulled down further for the body's oscillation to remain harmonic is
  - $mg/k$
  - $2mg/k$
  - $2k/mg$
  - $mg/2k$
- For a particle executing SHM, the kinetic energy is given by  $k = k_0 \cos^2 \omega t$ . The maximum value of potential energy is
  - $k_0$
  - $0$
  - $k_0/2$
  - none of these
- Two pendulums have time periods  $T$  and  $5T/4$ . They start executing SHM at the same time from the mean position. What will be the phase difference between them after the bigger pendulum completes one oscillation?
  - $45^\circ$
  - $90^\circ$
  - $60^\circ$
  - $30^\circ$
- The average acceleration in one time period in a SHM is
  - $a\omega^2$
  - $a\omega^2/2$
  - $a\omega^2/\sqrt{2}$
  - $0$
- A horizontal platform with a block of mass  $m$  placed on it is executing SHM along a vertical line. The amplitude of oscillation is  $0.25$  cm. What should be the maximum frequency of oscillation of the platform, if the block is not to be detached from the platform? (take  $g = 10 \text{ m/s}^2$  and  $\pi^2 = 10$ )
  - $10\pi \text{ s}^{-1}$
  - $10\pi \text{ s}^{-1}$
  - $20\pi \text{ s}^{-1}$
  - $10 \text{ s}^{-1}$
- Two pendulums of length  $121$  cm and  $100$  cm start vibrating at the same instant. They are in the mean position and in the same phase. After how many vibrations of the shorter pendulum, the two will be in the same phase in the mean position?
  - 10 vibrations
  - 11 vibrations
  - 21 vibrations
  - 20 vibrations
- A simple pendulum is made of a bob, which is a hollow sphere containing mercury, suspended by means of a wire. If mercury is drained off continuously the period of the pendulum will
  - remain unchanged
  - first increase, then decrease
  - decrease
  - become erratic
- Which of the following statement is not true?
 

In the case of a simple pendulum for small amplitude, the period of oscillation is

- a) directly proportional to the square root of the length of the pendulum  
 b) inversely proportional to square root of the length of the pendulum  
 c) dependent on the mass, size and material of the bob  
 d) independent of the amplitude
17. A particle is executing SHM along x-axis given by  $x = A \sin \omega t$ . The average velocity during the time interval  $t = 0$  to  $t = T$  is  
 a)  $4A/T$       b) 0      c)  $2A/T$       d)  $A/T$
18. A particle executes simple harmonic motion between  $x = -A$  and  $x = +A$ . The time taken for it to go from 0 to  $A/2$  is  $T_1$  and to go from  $A/2$  to  $A$  is  $T_2$ . Then  
 a)  $T_1 < T_2$       b)  $T_1 > T_2$       c)  $T_1 = T_2$       d)  $T_1 = 2T_2$
19. A simple pendulum with a brass bob has a time period  $T$ . The bob is now immersed in a non-viscous liquid and made to oscillate. The density of the liquid is  $(1/8)$ th that of the brass. The time period of the pendulum will be  
 a)  $\sqrt{8/7} T$       b)  $8/7T$       c)  $8^2/7^2 T$       d)  $T$
20. A particle is vibrating in SHM with an amplitude of 4 cm. At what displacement from the equilibrium position its energy is half potential and half kinetic?  
 a) 2cm      b)  $\sqrt{2}$  cm      c) 2 cm      d)  $2\sqrt{2}$  cm
21. The length of a simple pendulum is increased by 5%. What is the percentage increase in its time period?  
 a) 10%      b)  $\sqrt{44}\%$       c) 20%      d) 2.5%
22. A mass attached to a spring oscillates with a period of 2 sec. If the mass is increased by 4 kg, the time period increases by one second. Assume that Hooke's law is obeyed, initial mass  $M$  was  
 a) 3.2 kg      b) 1 kg      c) 2 kg      d) 8 kg
23. A particle of mass  $m$  executing SHM makes  $f$  oscillation per second. The difference of its kinetic energy when at the centre, and when at a distance  $x$  from the centre is  
 a)  $\pi^2 f^2 x^2 m$       b)  $2\pi^2 f^2 x^2 m$       c)  $1/2 \pi^2 f^2 x^2 m$       d)  $f^2 x^2 m$

#### MULTI CHOICE MULTI CORRECT

1. The displacement of a particle of mass 100 gm from its mean position is given by  $y = 0.05 \sin 4\pi(5t + 0.4)$ . Then  
 a) the time period of motion is 0.1 s.  
 b) the maximum acceleration of the particle is  $10\pi^2 \text{ m/s}^2$ .  
 c) Total energy of oscillation of the particle is  $0.05 \pi^2 \text{ joule}$ .  
 d) the force acting on the particle is zero when displacement is 0.05m.
2. A linear harmonic oscillator of force constant  $2 \times 10^6 \text{ Nm}^{-1}$  has a total mechanical energy of 160 J. its  
 a) maximum potential energy is 100J      b) maximum kinetic energy may 160 J.  
 c) maximum potential energy is 160 J      d) minimum potential energy is zero
3. If a SHM is given by  $y = (\sin \omega t + \cos \omega t)m$ , which of the following statement is true?  
 a) The amplitude is 1m.      b) The amplitude is  $\sqrt{2}m$ .      c) Time period is  $2\pi/\omega$ .  
 d) Time is considered from  $y = 0m$ .
4. A student performs an experiment for determination of Young's modulus of the material of a wire. He obtains the graph shown in the figure from his readings. The quantities on x and y axes may be, respectively:  
 a) weight suspended and increase in length  
 b) stress and increase in length  
 c) stress and strain developed  
 d) increase in length and weight suspended

#### NUMERICAL BASED TYPE

1. A particle in SHM crosses its equilibrium position at time  $t = 2$  sec. When the motion advances by one second more, its velocity is found to be  $\pi/3 \text{ m/s}$ . If the frequency of the motion is  $1/6 \text{ s}^{-1}$ , find the amplitude of the motion in meter.
2. A uniform rod of length  $L$  and mass 24 kg is pivoted at the centre. Its two ends are attached to two springs of equal spring constants 100 N/m. The springs are fixed to rigid supports as shown in the figure, and rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle  $\theta$  in one direction and released. Find the frequency of oscillation.

#### SECTION – II

1. The displacement of a particle at any instant  $t$  is represented by the equation  $y = A \sin \omega t + B \cos \omega t$ . The motion of particle is  
 a) SHM with amplitude  $A$  and  $B$       b) SHM with amplitude  $A^2 + B^2$   
 c) oscillatory but not SHM      d) SHM with amplitude  $\sqrt{A^2 + B^2}$
2. A particle performs SHM with time period  $T$ . The time taken by the particle to move from half the amplitude to the maximum displacement is  
 a)  $T/2$       b)  $T/4$       c)  $T/6$       d)  $T/8$
3. Two pendulum begin to swing simultaneously. During first 15 oscillations of the first pendulum, the other pendulum makes only 10 swings. The ratio between the lengths of these pendulums is

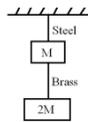
- a) 4/9                      b) 2/3                      c) 5/9                      d) 2/15

4. A particle moves in x-y plane according to the equation  $r = (i+2j)A \cos \omega t$ . The motion of the particle is

- a) simple harmonic                      b) uniformly accelerated                      c) circular motion  
d) projectile motion

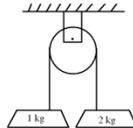
5. If the ratios of lengths, radii and Young's modulus of steel and brass wires in the adjacent figure are a, b and c, respectively, then the corresponding ratio of increase in their lengths would be

- a)  $2a^2c/b$                       b)  $3a/2b^2c$                       c)  $2ac/b^2$                       d)  $3c/2ab^2$



6. Two blocks of masses 1 kg and 2 kg are connected by a metal wire going over a smooth pulley as shown in the adjacent figure. The breaking stress of the metal is  $40/3\pi \times 10^6 \text{ N/m}^2$ . If  $g = 10 \text{ m/s}^2$ , then what should be the minimum radius of the wire used if it is not to break?

- a) 0.5 mm                      b) 1mm                      c) 1.5 mm                      d) 2mm



7. A 2m long light metal rod AB is suspended from the ceiling horizontally by means of two vertical wires of equal length, tied to its ends. One wire is of brass and has cross-section of  $0.2 \times 10^{-4} \text{ m}^2$  and the other is of steel with  $0.1 \times 10^{-4} \text{ m}^2$  cross-section. In order to have equal stresses in the two wires, a weight should be hung from the rod from end A at a distance of

- a) 66.6 cm                      b) 133 cm                      c) 44.4 cm                      d) 155.6 cm

8. Two points masses of 3 kg and 1 kg are attached to opposite ends of a horizontal spring whose spring constant is  $300 \text{ Nm}^{-1}$ . The angular frequency of the system is

- a) 40 rad/s                      b) 30 rad/s                      c) 20 rad/s                      d) 10 rad/s

9. The displacement y of a particle executing periodic motion is given by  $y = A \cos^2(1/2t) \sin(3t/2)$ . The motion constitutes superposition of n independent SHM, where n is

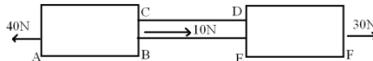
- a) 2                      b) 3                      c) 4                      d) 5

10. A particle of mass 5 gm is placed in a potential field. Its value is given by  $(5x^2+10)$  joule/kg. The frequency of oscillation of the particle is

- a)  $\sqrt{1000}$                       b)  $\sqrt{10/\pi}$                       c)  $\sqrt{10/2\pi}$                       d) insufficient information

11. Find the stress in CD. Area of  $CD = 2 \text{ m}^2$

- a)  $15 \text{ N/m}^2$                       b)  $5 \text{ N/m}^2$                       c)  $20 \text{ N/m}^2$                       d) none of these



12. Two wires A and B are of the same material. Their length are in the ratio 1:2 and diameter are in the ratio 2:1. If they are pulled by the same force, their increase in the length will be in the ratio

- a) 2:1                      b) 1:4                      c) 1:8                      d) 8:1

13. A simple pendulum with a solid metal bob has a period T. The metal bob is now immersed in a liquid of density one tenth that of the bob. The liquid is non-viscous. Now the period of the same pendulum with its bob remaining all the time in the liquid will be

- a) T                      b)  $9/10 T$                       c)  $T\sqrt{9/10}$                       d)  $T\sqrt{10/9}$