



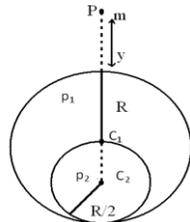
# THE GURUKUL INSTITUTE

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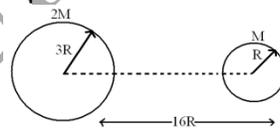
## PHYSICS- GRAVITATION

### Newton's Law Of Gravitation

1. Newton's apple fell towards the earth; didn't the earth move towards the apple?
2. Three identical particles, each of mass  $m$ , are placed at the three corners of an equilateral triangle of side ' $a$ '. Find the force exerted by this system on another particle of mass  $m$  placed at
  - a) the mid point of a side
  - b) the centre of the triangle.
3. On a sphere of radius  $R/2$  and density  $\rho_2$  with centre at  $C_2$  a second sphere is moulded with density  $\rho_1$  radius  $R$  and centre  $C_1$ . Find the force experienced by a point mass  $m$  at point  $P$  at a distance  $y$  from the combination as shown.



4. Somebody says "I weigh less than what I exactly weigh". Is he right? Is there any place on earth where he can weigh exactly?
5. Two spherical bodies of masses  $2M$  and  $M$  and of radii  $3R$  and  $R$ , respectively, are held at a distance  $16R$  from each other in free space. When they are released, they start approaching each other due to the gravitational force of attraction. Then, find:
  - a) the ratio of their acceleration during their motion
  - b) their velocities at the time of impact.



6. At a point above the surface of earth, the gravitational potential is  $-5.12 \times 10^7$  J/kg and the acceleration due to gravity is  $6.4 \text{ m/s}^2$ . Assuming the mean radius of the earth to be 6400 km, calculate the height of this point above the earth's surface.

### ESCAPE VELOCITY

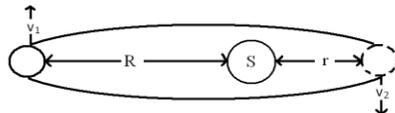
7. (i) What will be the escape velocity of a body if it is projected at an angle of  $45^\circ$  to the horizontal?  
(ii) Why are the lighter gases rare on the surface of earth?  
(iii) If a projectile is fired straight up from the earth's surface, what will happen if the total mechanical energy is (a) less than zero, and (b) greater than zero? [Ignore the air resistance and effects of other heavenly bodies.]
8. The mass of Jupiter is 318 times that of earth, and its radius is 11.2 times the earth's radius. Estimate the escape velocity of a body from Jupiter's surface. [ Given: The escape velocity from the earth's surface is  $11.2 \text{ km/s}$ ]

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9. Find the escape speed from a point at a height of  $R/2$  above the surface of earth as  $M$  and its radius as  $R$ .
10. Is it necessary for the plane of a satellite motion to pass through the centre of the earth?
11. A satellite is orbiting around earth. The centripetal force on the satellite is  $F$ . The gravitational force of earth on the satellite is also  $F$ . What is the net force on the satellite?
12. An artificial satellite of mass  $100 \text{ kg}$  is in circular orbit at  $500 \text{ km}$  above the earth's surface. Take the radius of the earth as  $6.5 \times 10^6 \text{ m}$ .
- a) Find the acceleration due to gravity at any point along the satellite path.
- b) What is the centripetal acceleration of the satellite?

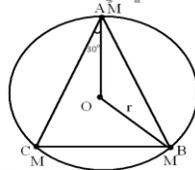
### KEPLER'S SECOND LAW

13. Calculate the mass of the Sun from the following data; distance between the Sun and Earth =  $1.49 \times 10^{11} \text{ m}$ ,  $G = 6.67 \times 10^{-11} \text{ SI units}$  and one year = 365 days.
14. A Saturn year is 29.5 times the earth year. How far is Saturn from the sun ( $M$ ) if the earth is  $1.5 \times 10^8 \text{ km}$  away from the sun?
15. A planet of mass  $m$  moves along an ellipse around the sun so that its maximum and minimum distance from the sun are equal to  $R$  and  $r$ , respectively. Find the angular momentum of this planet relative to the centre of the sun.



### PROBLEMS

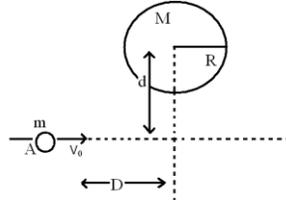
1. Three particles, each of mass  $M$ , are located at the vertices of an equilateral triangle of side ' $a$ '. At what speed must they move if they all revolve under the influence of their gravitational force of attraction in a circular orbit circumscribing the triangle while still preserving the equilateral triangle?



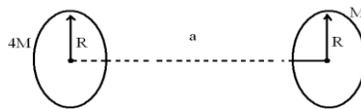
2. There are two fixed heavy masses of magnitude  $M$  of high density on  $x$ -axis at  $(d, 0, 0)$  and  $(-d, 0, 0)$ . A small mass  $m$  moves in a circle of radius  $R$  about origin in the  $y$ - $z$  plane between the heavy masses. Find the speed of the small particle.
3. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth.
- a) Determine the height of the satellite above the earth's surface.
- b) If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed with which it hits the surface of the earth. ( $g = 9.8 \text{ m/s}^2$  and  $R_E = 6400 \text{ km}$ .)
4. What is the gravitational potential energy of a particle of mass  $m$  kept at a distance  $x$  from the centre of a disc of mass  $M$  on its axis? The radius of the disc is  $R$ .
5. Two satellites of same mass are launched in the same orbit around the earth so that they rotate opposite to each other. If they collide elastically, obtain the total energy of the system before and just after the collision. Describe the subsequent motion of the wreckage.
6. A satellite of mass  $2 \times 10^3 \text{ kg}$  has to be shifted from an orbit of radius  $2R$  to another orbit of radius  $3R$ , where  $R$  is the radius of earth. Calculate the minimum energy required. [ $R = 6400 \text{ km}$  and  $g = 10 \text{ m/s}^2$ ]
7. Two satellites A and B of equal mass move in the equatorial plane of earth close to the earth's surface. Satellite A moves in the same direction as that of rotation of the earth while satellite B moves in the

opposite direction. Determine the ratio of the kinetic energy of B to that of A in the reference frame fixed to earth.

8. With what speed  $v_0$  should a body be projected as shown in the figure, with respect to a planet of mass  $M$  so that it would just be able to graze the planet and escape? The radius of the planet is  $R$ . (Assume that the planet is fixed).



9. Distance between the centres of two rings is  $a$ . The masses of these rings are  $M$  and  $4M$ . A body of mass  $m$  is fired straight from the centre of the heavier ring. What should be its minimum initial speed to cross the centre of lighter ring? The radius of the both ring is  $R$ . (Assume  $a \gg R$ ).



10. A uniform sphere has a mass  $M$  and radius  $R$ . Find the gravitational pressure  $P$  inside the sphere, as a function of the distance  $r$  from its centre.

### OBJECTIVE

1. A planet has twice the density of earth but the acceleration due to gravity on its surface is exactly the same as that on the surface of earth. Then, its radius in terms of the radius of earth ( $R$ ) will be:

- a)  $R/4$       b)  $R/2$       c)  $R/3$       d)  $R/8$

2. The period of rotation of the earth so as to make any object weightless on its equator is:

- a) 84 minutes      b) 74 minutes      c) 64 minutes      d) 54 minutes

3. Three particles, each of mass  $m$ , are placed at the corner of an equilateral triangle of side  $d$ . The potential energy of the system is:

- a)  $3Gm^2/d$       b)  $Gm^2/d$       c)  $-3Gm^2/d$       d) none of these

4. The minimum energy required to remove a body of mass  $m$  from earth's surface to far away is equal to:

- a)  $2mgR$       b)  $mgR$       c)  $-mgR$       d) zero

5. The gravitational field due to a mass distribution at position  $x$  is given by  $I = (A/x^3)$  in  $X$ -direction. The gravitational potential at position  $x$  is equal to:

- a)  $-A/x^3$       b)  $-A/2x^2$       c)  $+A/x^3$       d)  $A/2x^2$

6. A body of mass  $m$  starts approaching from far away towards the centre of a hypothetical hollow planet of mass  $M$  and radius  $R$ . The speed of the body when it passes the centre of the planet through its diametrical hole is:

- a)  $\sqrt{GM/VR}$       b)  $\sqrt{2GM/VR}$       c) zero      d) none of these

7. The energy required to shift a satellite from orbital radius  $r$  to orbital radius  $2r$  is  $E$ . What energy will be required to shift the satellite from orbital radius  $2r$  to orbital radius  $3r$ ?

- a)  $E$       b)  $E/2$       c)  $E/3$       d)  $E/4$

8. A satellite goes along an elliptical path around the earth. The rate of change of arc length 'a' swept by the satellite is proportional to:

- a)  $r$       b)  $r^2$       c)  $r^{1/2}$       d)  $r^{-1}$

9. Imagine a light planet revolving around a very massive star in a circular orbit of radius  $R$  with a period of revolution  $T$ . If the gravitational force of attraction between the planet and the star is proportional to  $R^{-5/2}$ , then  $T^2$  is proportional to:

- a)  $R^3$                       b)  $R^{7/2}$                       c)  $R^{3/2}$                       d)  $R^{3.75}$

## ASSIGNMENT

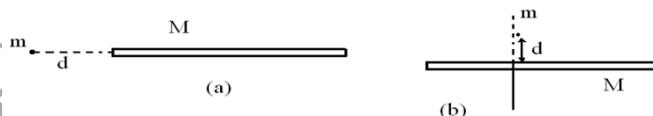
### Section – I

#### Level –I

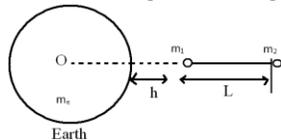
- Can an artificial satellite be put into orbit in such a way that it will always remain directly over New Delhi?
- Does the escape velocity of a body from the earth depend upon (a) the mass of the body, and (b) the direction of projection?
- How much quicker than at present must the earth revolve on its axis to make bodies at the equator experience weightlessness? What will be the duration of day then?
- How will weight of a body change at a height equal to earth's radius?
- Find the relation between the acceleration due to gravity  $g$  and the mean density  $\rho$  of earth in terms of  $G$  (the gravitational constant) and  $R_e$  (the radius of earth).
- The masses and radii of earth and moon are  $M_1, R_1$  and  $M_2, R_2$ , respectively. Their centres are at a distance  $d$  apart. Find the minimum speed with which a particle of mass  $m$  should be projected from a point midway between the two centres so as to escape to infinity.
- If a graph is plotted between  $T^2$  and  $r^3$  for a planet, then what will be the value of the slope of the graph? (Letters have usual meanings.)
- Two small bodies of masses 10 kg and 20 kg are kept a distance 1.0 m apart and released. Assuming that only mutual gravitational forces are acting, find the speeds of the particles when the separation decreases to 0.5 m.
- The gravitational field in a region is given by  $E = (2i + 3j)$  N/kg. Show that no work is done by the gravitational field when a particle is moved on the line  $3y + 2x = 5$ .
- A body is weighed by a spring balance to be 1.000 kg at the north pole. How much will it weigh at the equator? Account for the earth's rotation only.

#### LEVEL-II

- Find the gravitational force of attraction between a particle of mass  $m$  and a uniform slender rod of mass  $M$  and length  $L$  for the two orientations shown in the figure below.

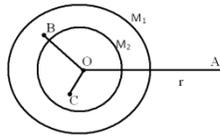


- Two bodies of masses  $m_1$  and  $m_2$  are connected by a long inextensible cord of length  $L$ . The combination is allowed to fall freely towards the earth (mass  $m_e$ ), the direction of the cord being always radial as shown in the figure. Find: (a) the tension in the cord and (b) the accelerations of  $m_1$  and  $m_2$ . Does the cord ever become slack? (ignore the gravitational interaction between  $m_1$  and  $m_2$ )

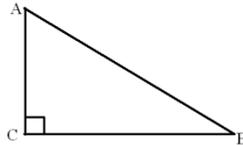


- Two concentric shells of masses  $M_1$  and  $M_2$  are situated as shown in Figure. Find the force on a particle of mass  $m$  when the particle is located at (a)  $r = a$  (b)  $r = b$  (c)  $r = c$ .

The distance  $r$  is measured from the centre of the shell.  
Given that  $OA=a$ ,  $OB=b$ ,  $OC=c$



4. Two small particles of mass  $m$  each are placed at the vertices A and B of a right angle isosceles triangle. If  $AB=L$ , find the gravitational field strength at C.



5. A body of mass  $m$  is taken to a height  $kR$  from the surface of the earth very slowly,  $R$  being the radius of the earth. Find the change in gravitational potential energy in this process. [Take  $m_e$  the mass of earth.]

6. A body is thrown up (radially outward from the surface of the earth) with a velocity equal to one-fourth of the escape velocity. Find the maximum height reached from the surface of the earth. (Radius of earth is  $R_e$ )

7. Four particles of equal mass  $M$  move along a circle of radius  $R$  under the action of their mutual gravitational attraction. Find the speed of each particle.

8. A body starts from rest from a point at a distance  $r_0 (> R_e)$  from the centre of the earth. It reaches the surface of earth. What is the velocity acquired by the body?

9. A space vehicle of mass  $m$  is in a circular orbit of radius  $2R_e$  about the earth (mass  $m_e$ ). What is the work done by an external agent to transfer it to an orbit of radius  $4R_e$ ?

10. Two massive particles of masses  $M$  and  $m (M > m)$  are separated by a distance  $L$ . They rotate with equal angular velocity under their gravitational attraction. What is the linear speed of the particle of mass  $m$ ?

11. A satellite is put in an orbit just above the earth's atmosphere with a velocity  $\sqrt{1.5}$  times the velocity for a circular orbit at that height. The initial velocity imparted is horizontal. What would be the maximum distance of the satellite from the surface of the earth when it is in the orbit?

12. Two satellite A and B revolve around a planet in coplanar circular orbit in the same direction with period of revolutions 1 hour and 8 hours respectively. The radius of satellite A is  $10^4$  km then find the angular speed of 'B' with respect to A?

13. Three stars each of mass  $M$  and radius  $R$  are initially at rest and the distance between centres of any two stars is  $d$  and they form an equilateral triangle. They start moving towards the centroid due to mutual force of attraction. What are the velocities of the stars just before the collision? Radius of each star is  $R$ .

14. The radius of a planet is  $R_1$  and a satellite revolves round it in a circle of radius  $R_2$ . The time period of revolution is  $T$ . Find the acceleration due to the gravitation of the planet at its surface.

## Part- B

### OBJECTIVE

#### ( Multi Choice Single Correct)

1. The weight of a body at the centre of the earth is

- a) zero                      b) infinite                      c) same as on the surface of earth  
d) none of these

2. Dimensional formula of 'G' is

- a)  $M^{-1}L^2T^2$                       b)  $M^2L^{-1}T^2$                       c)  $M^{-1}L^2T^2$                       d)  $M^{-1}L^{-2}T^{-1}$

3. Infinite number of bodies, each of mass 6 kg, are situated at distances 1m, 2m, 4m, 8m,....., from the origin on the y-axis. The resultant gravitational field intensity at the origin is

- a) 4G                      b) 8G                      c) 9G                      d) 12G

4. The ratio of the radius of earth to that of the moon is 10. The ratio of the acceleration due to gravity on the earth to that on the moon is 6. The ratio of the escape velocity from the earth's surface to that from the moon will be

- a) 4                      b) 6                      c) 7.75                      d) 12

5. The value of acceleration due to gravity at a point P inside the earth and at another point Q outside the earth is  $g/2$ . (  $g$  being acceleration due to gravity at the surface of earth.) Maximum possible distance in terms of radius of earth  $R$  between P and Q is:

- a)  $2R$                       b)  $2R(\sqrt{2}+1)$                       c)  $R/2 \times (2\sqrt{2}-1)$   
d)  $R/2 \times (2\sqrt{2}+1)$

6. The earth revolves round the sun in an elliptical orbit. Its speed

- a) goes on decreasing continuously  
b) is greatest when it is closest to the sun  
c) is greatest when it is farthest from the sun  
d) is constant at all the points on the orbit

7. Two satellites are orbiting around the earth in circular orbits of same radii. One of them is 10 times greater in mass than the other. Their periods of revolutions are in the ratio.

- a) 100:1                      b) 1:100                      c) 10:1                      d) 1:1

8. An earth satellite is moving round the earth in a circular orbit. For such a satellite, which of the following statement is wrong?

- a) It is a freely falling body.  
b) It is moving with a constant speed.  
c) Its acceleration is zero.  
d) Its angular momentum remains constant.

9. An earth satellite has a mass  $M$  and angular momentum  $L$ . Its areal velocity is given by:

- a)  $(L/M)$  m/s                      b)  $(2L/M)$  m<sup>2</sup>/s                      c)  $(L/2M)$  m<sup>2</sup>/s  
d)  $(L/2M)$  m/s

10. Let  $V$  and  $E$  be the gravitational potential and gravitational field at a distance  $r$  from the centre of a uniform spherical shell. Consider the following two statements:

- A) The plot of  $V$  against  $r$  is discontinuous, and  
B) The plot of  $E$  against  $r$  is discontinuous.

Now, select the correct option from the following.

- a) Both (A) and (B) are correct.  
b) (A) is correct but (B) is incorrect.  
c) (B) is correct but (A) is incorrect.  
d) Both (A) and (B) are incorrect.

11. If a body is projected with a speed less than the escape velocity,

- a) it must reach a certain height and may fall down following a straight path.  
b) it must reach a certain height and may fall down following a parabolic path  
c) it may orbit the earth in a circular or elliptical orbit  
d) it must orbit the earth in a circular orbit

12. A particle of mass  $m_1$  lies inside a spherical shell of mass  $m_2$  and radius  $R$  at a distance  $r$  from the centre. The gravitational potential energy of the system is:

- a)  $-Gm_1m_2/r$                       b)  $-Gm_1m_2/R$                       c)  $+Gm_1m_2/r$   
d)  $+Gm_1m_2/R$

13. If the radius of the earth shrinks by 1%, its mass remaining same, the acceleration due to gravity on the surface of the earth will

- a) decrease by 2%                      b) decrease by 0.5%  
c) increase by 2%                      d) increase by 0.5%

14. A satellite is launched into a circular orbit of radius  $R$  around the earth. A second satellite is launched into an orbit of radius  $1.01R$ . The time period of the second satellite is larger than that of the first one by approximately

- a) 0.5%                      b) 1.5%                      c) 1%                      d) 3.0%

15. An artificial satellite, moving in a circular orbit around the earth, has a total energy (K.E.+P.E.).  $E_0$  Its potential energy is

- a)  $-E_0$                       b)  $1.5E_0$                       c)  $2E_0$                       d)  $E_0$

16. The time period of revolution of a satellite is  $T$ . The kinetic energy of the satellite is proportional to

- a)  $T$                       b)  $T^2$                       c)  $T^3$                       d)  $T^{-2/3}$

17. A double star is a system of two stars (say having masses  $m_1$  and  $m_2$ ) moving around the centre of inertia of the system due to gravitation. Then, ratio of sweeps of area of star of mass  $m_1$  to the star of mass  $m_2$  is

- a)  $m_1/m_2$                       b)  $m_2/m_1$                       c)  $m_1^2/m_2^2$                       d)  $m_2^2/m_1^2$

18.  $R$  is the radius of earth,  $\omega$  is its angular velocity and  $g_p$  is the value of  $g$  at poles. The effective value of  $g$  at the latitude  $\lambda=60^\circ$  will be equal to

- a)  $g_p - 1/4 R\omega^2$                       b)  $g_p - 3/4 R\omega^2$                       c)  $g_p - R\omega^2$                       d)  $g_p + 1/4 R\omega^2$

19. When a body is taken from equator to the poles, its weight

- a) remains the same                      b) increases                      c) decreases  
d) increases at north pole and decreases at south pole

20. A planet has twice the density of earth but the acceleration due to gravity on its surface is exactly the same as on the surface of the earth. Its radius in terms of radius of the earth  $R$  will be

- a)  $R/4$                       b)  $R/2$                       c)  $R/2$                       d)  $R/8$

21. If the acceleration due to gravity at the surface of the earth is  $g$ , the work done in slowly lifting a body of mass  $m$  from the earth's surface to a height  $R$  equal to the radius of the earth is

- a)  $1/2 mgR$                       b)  $2mgR$                       c)  $mgR$                       d)  $1/4 mgR$

22. A person brings a mass of 1 kg from infinity to a point A. Initially the mass was at rest but it moves at a speed of 2 m/s as it reaches A. The work done by the person on the mass is  $-2J$ . The potential at A is

- a)  $-2J/kg$                       b)  $-2J/kg$                       c)  $-5J/kg$                       d) none of these

23. If a satellite be rotating about a planet, which of the following is true?

[ $U$ = Potential energy of planet satellite system,  $K$ = kinetic energy of satellite,  $T$ = Total Mechanical energy]

- a)  $|U| = K = |T|$                       b)  $K = |T| = |2U|$                       c)  $2K = |T| = |U|$   
d)  $K = |T| = |1/2U|$

24. A person sitting in a chair in a satellite feels weightless because

- a) The earth does not attract the object in a satellite  
b) the normal force by the chair on the person balances the earth's attraction.  
c) the normal force is zero  
d) the person in satellite is not accelerated.

## MULTI CHOICE MULTI CORRECT

1. In case of earth:

- a) Potential is minimum at the centre of earth.
- b) Potential is same, both at centre and infinity but not zero.
- c) Potential is zero, both at centre and infinity
- d) Field is zero, both at centre and infinity

2. Two satellite of same mass of a planet in circular orbits have period of revolutions 32 days and 256 days. If the orbital radius of the first is  $R$ , then

- a) the kinetic energy of the second is less than that of the first
- b) the total mechanical energy of the second is greater than that of the first
- c) Radius of the orbit of second is  $4R$
- d) Radius of the orbit of second is  $8R$

3. Gravitational field due to a point mass is

- a) A central field
- b) always pointed towards the mass
- c) an inverse square field
- d) a conservative field

4. Which of the following statement(s) is/are correct?

- a) An astronaut going from earth to moon will experience weightless once.
- b) When a thin uniform spherical shell gradually maintaining its shape, the gravitational potential at the centre increases.
- c) In the case of spherical shell, the plot of potential versus distance from centre is continuous.
- d) In the case of spherical shell, the plot of gravitational field intensity  $I$  versus distance from centre is continuous.

5. Two satellite A and B move around the earth in a circular orbit. The mass of B is twice the mass of A then

- a) speeds of A and B are equal
- b) kinetic energy are equal
- c) kinetic energy of B is greater than A
- d) kinetic energy of A is greater than B

6. Choose the wrong statement. The radius of the orbit of a geostationary satellite depends upon

- a) mass of the satellite, its time period and the gravitational constant
- b) mass of earth, mass of satellite, time period of satellite and gravitational constant
- c) mass of earth, time period of the satellite and the gravitational constant
- d) none of the above

## Numerical Based Type

1. A pendulum clock which keeps correct time at the bottom of mountain loses 27 second/day when it is taken to the top of mountain. Find the height of mountain (in km)? ( $R_e = 6400\text{ km}$ )

2. A body stretches a spring by a particular length at the earth's surface at equator. At height  $3\lambda$  km above the south pole it will stretch the same spring by the same length. Assume the earth to be spherical. (take  $\omega^2 R = 0.0375$ , where  $\omega$  and  $R$  have usual meanings). Find  $\lambda$ .

3. A mass equal to the mass of the earth is to be compressed in a sphere in such a way that the escape velocity from its surface is  $3 \times 10^8$  m/s. what should be the radius of the sphere in mm? (take  $GM_E = 40.5 \times 10^{13} \text{ Nm}^2 \text{ kg}^{-1}$ )