THE GURUKUL INSTITUTE

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D.P.P of Mathematics – AREA

Working Rule For Finding The Area

1.Find the area bounded by $y = \cos x$, $x = -\pi/2$, $x = 2\pi$ and the x – axis.

2. Calculate the area bounded by $y= x^2$ on the left of y – axis, the y – axis and the lines y=1, y=4.

3.Find the area included between the curves $y= \sin^{-1} x$, $y= \cos^{-1}x$ and the x – axis.

4.Compute the area of the region bounded by the straight lines x=0, x=2 and the curves $y=2^x$, $y= 2x – x^2$.

5.Find the area bounded by $y= x|\sin x|$ and the x – axis between x=0, $x=2\pi$.

6.Find the area bounded by the curve $|x|+y=1$ and the x – axis.

7.Calculate the area bounded by the curve $y= x(3 – x)^2$, the x – axis and the ordinates of the maximum and minimum points of the curve.

8.Let f be a real valued function satisfying $f(x/y)= f(x) – f(y)$ and $\lim f(1+x)/x = 3$, then find the area bounded by y = f(x), y – axis and line y=3,

9. Let $f(x)=$ minimum $(x+1, \sqrt{1-x})$ for all $x\leq 1$. Find the area bounded by y= f(x) and the x – axis.

PROBLEMS

1.Find the area bounded by the curves $y= -\sqrt{4-x^2}$, $x^2= -\sqrt{2}\,y$ and x=y.

2.If the line y=mx divides the area enclosed by the lines x=0 ,y=0, x= 3/2 and the curve $y= 1+4x – x^2$ into two equal parts, then find the value of m.

3.A curve y= f(x) passes the origin and lines entirely in the first quadrant. Through any point (x,y) on the curve, lines are drawn parallel to the coordinate axis. If the curve divides the area formed by these lines and the coordinate axes in the ratio m:n, find f(x).

4.Find the area enclosed by the curve y= f(x) where f(x) is the polynomial function of least degree satisfying $\lim 1+\left[\frac{f(x)}{x^3}\right]^{1/x} = e$ and the circle $x^2+y^2=2$ above the x – axis.

5.Find the area enclosed by $y=x^2+ \cos x$ and its normal at $x= \pi/2$ in the first quadrant.

6.Find the ratio of areas in which the curve $y =\left[\frac{x^3}{100} + \frac{x}{35}\right]$ divides the circle $x^2+y^2 – 4x+2y+1=0$. ([.] denotes greatest integer function).

7.Find the area of the region enclosed by the curve $5x^2+6xy+2y^2+ 7x+ 6y+6=0$.

8.Consider a square with vertices at (1,1),(-1,1),(1,-1) and (-1,-1). Let S be the region consisting of all points inside the square which are nearer to the origin than to any edges. Sketch the region S and find its area.

OBJECTIVE

1.The area bounded by the curve $y=x^3$, the x – axis and the ordinates x= - 2 and x=1 is

a) – 9 b) – 15/4 c) 15/4 d) 17/4

2.The maximum area of a rectangle whose two vertices lie on the x – axis and two on the curve $y= 3 - |x|$, $-3\leq x\leq 3$ is

a) 9 b) 9/2 c) 3 d) none of these

3.The curve $x=4 – 3y – y^2$ cuts the y – axis into two points P and Q. Then the area enclosed by the y – axis and the portion of the curve which lies between P and Q is

a) 20 sq. units b) 18 sq. units c) 17 sq. units d) none of these

4.The curve $y= x^2 – 7x+10$ intersects the x – axis at the points A and B. Then the area bounded by the curve and the line AB is

a) $4 \times 1/2$ sq. units b) 4 sq. units c) 6 sq. units d) 2 sq. units

5.The area of the region bounded by the curve y= $\frac{1}{1+(\tan x)^{1/2}}$ and the x – axis between the ordinates x= π/6 and x= π/3 is

a) π/4 b)π/2 c) π/8 d) none of these

6. The area bounded by y = ln x, the x – axis and the ordinates x=0 and x=1, is

a) 1 b) 3/2 c) -1 d) none of these

7.The area enclosed by the parabola $ay= 3(a^2 – x^2)$ and the x – axis is

a) $4a^2$ sq. units b) a^2 sq. units c) $6a^2$ sq. units d) $5a^2$ sq. units

8.The area between the curves $y= xe^x$ and $y\ xe^{-x}$ and the line x=1 is

a) 2e b) e c) 2/e d) 1/e

9. The area in the first quadrant bounded by $y= 4x^2$, x=0 y=1, and y=4 is

a) 2 sq. units b) 2×1/2 sq. units c) 2×1/3 sq. units d) 3 sq. units

10. The slope of tangent to a curve y= f(x) at (x,f(x)) is 2x+1. If the curve passes through the point (1,2), then the area bounded by the curve , x – axis and line x=1 is

a) 5/6 b) 6/5 c) 1/6 d) 6

EXERCISE – 1

1.Find the area enclosed by |x|+|y| =1.

2.Find the area of the region bounded by C: y=tan x, tangent drawn to C at x = π/4 and the x – axis.

3.Find the area bounded by $y^2=4ax$ and the tangents at the ends of its latus rectum.

4.Calculate the area bounded by the curve y(y – 1)=x and the y – axis.

5.Find the point P on the parabola $y^2= 4ax$ such that area bounded by the parabola, the x – axis and the tangent at P is equal to that of bounded by the parabola, the x – axis and the normal at P.

6.Find the area bounded by the curve y=(x – 1) (x – 2) (x – 3), the x – axis and lying between the ordinates x=0 and x=3.

7. Area bounded by y= [sin x+ cos x] and the lines x=0,x=π is (where [.] denotes the greatest integer function)

a) π b) π/2 c) 3π/2 d)3π/4

8. Area enclosed between the parabola $y^2=4ax$ and $x^2 = 4ay$ is

a) $16a^2/3$ b) $32a^2/3$ c) $8a^2/3$ d) $24a^2/3$

9.Area bounded by y = [tan x] with x – axis between x =0 and x= π/3 is (where [.] denotes the greatest integer function)

a) π/12 b)π/6 c) 2π/3 d) none of these

10.Area bounded by y= ln x and $y = (\ln x)^2$ is

a) 3 – e b) e c) 2e – 3 d) e+1

EXERCISE – 2

1. Find the area bounded by y=x – 1, the x – axis and the ordinates x=1, x=3.

2.Find the area common to the circle $x^2+y^2= 16a^2$ and the parabola $y^2 = 6ax$, a>0.

3.Prove that area common to the parabolas $y= 2x^2$ and $y= x^2+4$ is 32/3 sq. units.

4.If the area bounded by y= f(x), the x – axis, the y – axis and x=t (t>0) is t^2, then find f(x).

5.Find the area of the region bounded by [x] and {x}, where [.] and {.} are greatest integer function and fractional part of x respectively.

6.Find the area bounded by the curves $y=x+ \sqrt{x^2}$ and xy= 1, the x – axis and the line x=2.

7.Find the area bounded by the curves y= ln |x|, y – axis and y=1 - |x|.

8.The area bounded by the curves y=ln x, y= |ln x| and the y – axis is

a) 3 b) 2 c) 4 d) 8

9.If area bounded by y= f(x), the coordinate axes and the line x=a is given by ae^a, then f(x) is

a) $\pm e^x(x+1)$ b) e^x c) $x e^x$ d) $x e^x+1$

10.The area common to the region determined by $y \geq \sqrt{x}$, and $x^2+y^2<2$ has the value
a) $\pi - 2$ b) $2\pi - 1$ c) $3\pi - \sqrt{2}/3$ d) none of these

MISCELLANEOUS EXERCISE

1.Find the area of the region included between the parabola $y = 3x^2/4$ and the line $3x - 2y+12=0$.
2.Compute the area of one of the curvilinear triangles bounded by the x – axis and the curves y= sin x and y= cosx.
3.Find the area of the region bounded by the parabolas $x= - 2y^2$, $x= 1 - 3y^2$ and the y – axis.
4.Find the included between the parabola $y= x^2/4a$ and the curve $y= 8a^3/x^2+4a^2$
5.An ellipse is cutout of radius a, the major axis of the ellipse coincides with one of the diameter of the circle while the minor axis is equal to 2b. Prove that the area of the remaining part equals that of the ellipse with the semi axes a and a – b.
6.Find the equation of the curve if the area bounded by the curve y= f(x), x – axis and the line x=a and x=b is equal to $\sqrt{b^2 - a^2}$ $\forall$ b>a (a is a constant).
7.Normals are drawn from the point (9,6) to the parabola $y^2=4x$. Then find the area enclosed by the parabola and the two normals other than the normal at (9,6).
8.Find the area bounded by the curves y=x and $y=x^3$.
9.Find the area bounded by the curve y=tan x, the y – axis and the straight lines y= tan a, and y= tan b (0<a, b< $\pi/2$).
10.(i) Compute the area of the region bounded by the curves y=tanx and $y= \tan^2 x$ ($-\pi/2 <x< \pi/2$).
(ii)Find the area bounded by the curve y= f(x) and x – axis between x=0 and x= $\pi/2$, where f(x) = min.(tan x, cot x, $1/\sqrt{3}$).
11.Find the area between the curve $y= 2x^4 - x^2$, the x – axis and the ordinates of two minima of the curve.
12.Find the area bounded by f(x)= maximum {sin x, cos x}, x=0, x=2π and the x – axis.
13. Find the area of the region bounded by the parabola $(y - 2)^2= x - 1$, the tangent to it at the point with ordinate $y_0= 3$ and the x – axis.
14.(i)Find the area bounded by y = f(x) and the curve $y= 2/(1+x^2)$ satisfying the conditions f(x).f(y)=f(xy) $\forall$ x, y ϵ R and f'(1)=2, f(1)=1.
(ii) Find the area bounded by the curve f(x)= maximum {1+ sin x, 1, 1 – cos x} and the x – axis between the ordinates x= - π and x=π.
15.If the area bounded by the curve, y= f(x), the lines x=1, x=b and the x axis is (b – 1) cos(3b+4), b>1, then f(x) is
a)(x – 5) sin (3x+4) b) (x – 1) sin (x+1)+(x+1) cos(x – 1) c) cos(3x+4) – 3(x – 1) sin (3x+4)
d) (x – 5) cos (3x+4)

ASSIGNMENTS
SECTION – I (PART – A)

1.Find the area of the partitions cut off by the hyperbola $x^2 - 3y^2=1$ from the ellipse $x^2+4y^2=8$.
2.Find the area of the region formed by $x^2+y^2 - 6x - 4y+12 \leq 0$, $y \leq x$ and $x \leq 5/2$.
3.A chord is drawn to the curve $x^2+2x - y+2=0$ at the point whose abscissa is 1, and it is parallel to the line y=x. Find the area in the first quadrant bounded by the curve,this chord and the y – axis.
4.Find the area between the curves $y=x^2+x - 2$ and y= 2x, for which, $|x^2+x - 2|+|2x| = |x^2+3x - 2|$ is satisfied.
5.(i) Show that the area included between the parabolas $y^2= 4a(x+a)$ and $y^2=4b(b - x)$ is 8/3 $\sqrt{ab}$ (a+b).
(ii) Find all the possible values of b>0, so that the area of the bounded region enclosed between the parabolas $y=x - bx^2$ and $y= x^2/b$ is maximum.

6. If a function f(x) is bijective in [a,b], then prove that $\int_a^b f(x)dx + \int_{f(a)}^{f(b)} f(x)dx$ = bf(b) – af(a).

7.For any real t, x=1/2($e^t + e^{-t}$), y=1/2($e^t - e^{-t}$) is a point on the hyperbola $x^2 - y^2=1$. Show that the area bounded by the hyperbola and the line joining its center to the points corresponding to t_1 and $-t_1$ is t_1.

8.The area enclosed by the curve f(x)=24+ax – x^2, co – ordinate axes and the ordinate x=6 is 108. If m and n are x – axis intercepts of the graph of y= f(x), then find the value of m+n+a.

9.Let f(x) be a polynomial of degree three such that the curve y= f(x) has relative extremes at x= $\pm$ 2/$\sqrt{3}$ and passes through (0,0) and (1,-1) dividing the circle $x^2+y^2=4$ in two parts. Find the ratio of the areas of two parts.

PART – B(Multi Choice Single Correct)

1.The area bounded by $y^2=2x+1$ and x – y – 1=0 is

a) 4/3 b) 8/3 c) 16/3 d) none of these

2.The area of the smaller region bounded by the circle $x^2+y^2=1$ and |y|= x+1 is

a) π/4 – 1/2 b) π/2 – 1 c) π/2 d) π/2+1

3.Area bounded by the curve [|x|] + [|y|] =3, where [.] denotes the greatest integer function

a) 8 sq. units b) 12 sq. units c) 16 sq. units d) 20 sq. units

4.The area bounded by the curve y= x^2+2x+1, the tangent at (1,4) and the y – axis is

a) 1 b) 1/2 c) 1/3 d) ¼

5.The total area enclosed by the lines y= |x|, |x|=1, and y=0 is

a) 1/2 b) 1 c) 3/2 d) 2

6.The area bounded by y= |x|/x, x ≠ 0 the y – axis and the curve $y=x^3$ is

a) 3√3 b) 3/2 c) 2/3 d) 2√3

7.The ration in which the area bounded by the curves $y^2=x$ and $x^2=y$ is divided by the line x= 1/2 is

a) (4√2 – 1):(9 - 4√2) b) (3√2+3):(9 - 4√2) c) (√2 – 1): (√3 – 1)

d)(2√2 – 1) : (3√3 – 1)

8.The area of bounded by the curve y=log x, the x – axis and the line x=e is

a) e sq.units b) (1 – 1/e) sq. units c) (2e – 1) sq. units d) 1 sq. units

9.The area of the region bounded by the curve x = $\sin^{-1}$ y, the x – axis and the lines |x| =1 is

a) 2 – 2cos 1 b) 1 – cos 1 c) 1 – 2cos 1 d) none of these

10.The area of the region bounded by the curves y= |x| and y=3 - |x| is

a) 9/4 b) 3 c) 9/2 d) 9

11.The area of the circle $(x – 2)^2 + (y – 3)^2=32$ below the line y= x+1 is

a) 32π/3 b) 32π c) 16π d) none of these

12.If A_m represents the area bounded by the curve y= In x^m, the x – axis and the lines x=1 and x=e, then A_m+ mA_{m-1} is

a) m b) m^2 c) $m^2/2$ d) $m^2 – 1$

MULTI CHOICE MULTI CORRECT

1.If the are bounded by the curve y= x – x^2 and line y = mx is equal to 9/2 sq. units, then m may be

a) -4 b) -2 c) 2 d) 4

2.If the area bounded by the curve y=sin 2x and lines x= π/6, x=a and x – axis is equal to 1/2, then a is

a) π/3 b) π/2 c) – π/6 d) π

3.If the parabola y= $x^2/2$ divides the circle $x^2+y^2=8$ into two parts, then the area of the parts may be

a) 6π+ 4/3 sq. units b) 2π – 4/3 sq.units c) π+ 4/3 sq. units

d) 6π – 4/3 sq. units

SECTION – II

(Multi Choice Single Correct)

1.The area common to $y^2=x$ and $x^2=y$ is

a) 1 b) 2/3 c) 1/3 d) none of these

2.The area of the region bounded by $y= |x – 1|$ and $y=1$ is

a)1/2 b)1 c) 2 d) none of these

3.The area bounded by the curve $y^2=9x$ and the lines $x=1, x=4$ and $y=0$, in the first quadrant, is

a) 7 b) 14 c)28 d) 14/3

4. The enclosed by $y=1$ and $\pm 2x+y=2$ is (in square unit)

a)1/2 b) 1/4 c) 1 d) none of these

5.The area of region bounded by the curve $y= 2x – x^2$ and the line $y=x$ is

a)1/2 b) 1/3 c) 1/4 d) 1/6

6.The area of the region bounded by $x^2=y$, $y=x+2$ and the x – axis is

a) 8 b) 4 c) 2 d) none of these

7.The area bounded by the curve $x= 6y – y^2$ and y – axis is

a) 25 sq. units b) 36 sq. units c) 16 sq. units d) 40 sq. units

8.The area enclosed by $y = \ln x$, its normal at (1,0) and the y – axis is

a) 1/2 b) 3/2 c) not defined d) none of these

9. Area bounded by ellipse $x^2/a^2 + y^2/b^2 = 1$ is

a) πab b) $\pi a^2 b$ c) πab^2 d) ab

10.Area bounded by $y^2 = -4x$ and its latus rectum is

a) 4/3 b) 8/3 c) 2/3 d) 16/3