



THE GURUKUL INSTITUTE

PLOT 5C, 2ND FLOOR, COMPLEX, SEC-13, OPP. JAIPURIA SCHOOL,
VASUNDHARA, GHAZIABAD (U.P). CELL: 9810780903

D .P .P of Mathematics- Permutation & Combination

Fundamental Principle Of Counting

1. A college offers 7 courses in the morning and 5 in the evening. Find the possible number of choices with the student who wants to study one course in the morning and one in the evening.
2. A person wants to go from station A to station C via station B. There are three routes from A to B and four routes from B to C. In how many ways he can travel from A to C?

Rule Of Sum

3. A college offers 7 courses in the morning and 5 in the evening. Find the number of ways a student can select exactly one course, either in the morning or in the evening.
4. A person wants to leave station B. There are three routes from station B to A and four route from B to C. In how many ways he can leave the station B.
5. How many (i) 5- digit, (ii) 3- digit numbers can be formed by using 1,2,3,4,5 without repetition of digits.
6. How many 4- letter words can be formed using a,b,c,d,e
(i) without repetition, (ii) with repetition.

Permutations

7. How many 7- letter words can be formed using the letters of the word:
(a) BELFAST, (b) ALABAMA
8. (a) How many anagrams can be made by using the letters of the word HINDUSTAN.
(b) In how many of these anagrams all the vowels come together.
(c) In how many of these anagrams none of the vowels come together.
(d) How many of these anagrams begin and end with a vowel.
(e) In how many of these anagrams do the vowels and the consonants occupy the same relative positions as in HINDUSTAN.
9. How many 3 digit numbers can be formed using the digits 0,1,2,3,4,5 where
a) digits may not be repeated, (b) digits may be repeated.
10. Find the number of ways in which 6 letters can be posted in 10 letterboxes.
11. Find the number of permutations that can be had from the letters of the word 'OMEGA' such that
(i) O & A occupy odd places, (ii) E always occupies the middle place
(iii) Vowels occupy odd places (iv) All the vowels are never together

Circular Permutation

12. 20 persons were invited to a party. In how many ways can they be seated in a round table such that two particular persons sit on either side of the host?
13. In how many ways 10 boys and 5 girls can sit around a circle table, so that no two girls sit together.
14. In how many ways can 20 persons be seated round a table if there are 9 chairs?
15. How many necklace of 10 beads each can be made from 20 beads of different colours?
16. (i) In how many different ways can five boys and five girls form a circle such that the boys and girls alternate?
(ii) Consider 23 different coloured beads on a necklace. How many ways can the beads be placed on this necklace such that 3 specific beads always remain together?

Combinations

17. Let 15 toys be distributed among 3 children subject to the condition that any child can take any number of toys. Find the required number of ways to do this if
a) toys are distinct, (b) if toys are identical.
18. A delegation of four students is to be selected from a total of 12 students. In how many ways can the delegation be selected if
(a) all the students are equally willing,
(b) two particular students have to be included in the delegation,
(c) two particular students do not wish to be together in the delegation,
(d) two particular students wish to be included together only,
(e) two particular students refuse to be together and two other particular students wish to be together only in the delegation.
19. (a) How many diagonals are there in an n - sided polygon ($n > 3$)?
(b) How many triangles can be formed by joining the vertices of an n - sided polygon?
How many of these triangles have
(i) exactly one side common with that of the polygon?

- (ii) exactly two sides common with that of the polygon?
 (iii) no side common with that of the polygon?

All Possible Selections

20. In how many ways can a person having 3 coins of 25 paise, 4 coins of 50 paise and 2 coins of 1 rupee give none or some coins to a beggar?
 21. How many positive factors are there of the number 360 and find the sum of all these factors.
 22. (i) How many non-prime factors are in the number $N = 2^5 \times 3^7 \times 9^2 \times 11^4 \times 13^3$.
 (ii) A candidate is required to answer 6 out of 10 questions which are divided into two groups, each containing 5 questions. If he is not permitted to attempt more than 4 questions from each group find the numbers of ways in which he can attempt the paper.

(Derangements)

23. Supposing 4 letters are placed in 4 different envelopes. In how many ways can they be taken out from their original envelopes and distributed among the 4 different envelopes so that no letters remain in its original envelope?

Multinomial Theorem

24. In how many different ways three persons A, B, C having 6, 7 and 8 one rupee coins respectively can donate Rs. 10- collectively.
 25. (i) In how many ways can 10 identical blankets be given to 3 beggars such that each receives at least one blanket?
 (ii) In how many ways can $2n$ people be divided into n pairs?
 (iii) In how many ways can 22 distinct books be given to 5 students so that two students have 5 books each and another three students have 4 books each?

PROBLEMS

1. In how many ways can the letters of the word CONCUBINE be arranged so that (a) the C's are never together, (b) C's are always together?
 2. Suppose a man has 5 aunts and 6 uncles and his wife has 6 aunts and 5 uncles. In how many ways can he call a dinner party of 3 men and women so that there are exactly 3 of the man's relatives and 3 of the wife's?
 3. (a) In how many ways mn things be equally distributed in n groups?
 (b) In how many ways can you equally distribute 100 packages of food to 10 refugees?
 (c) Find the number of ways of selecting r pairs out of n different things.
 4. In how many ways can we select four cards of an ordinary pack of playing cards so that exactly three of them are of the same denomination.
 5. The sides AB, BC, CA of a triangle ABC have respectively 3, 4 and 5 points lying on them. Find the number of triangles that can be formed using these points as vertices.
 6. Find the number of ways in which 18 identical balls can be used in 15 different cricket matches.
 7. How many 3 letter words can be formed from the letters of the word CALCUTTA?
 8. In how many ways 3 boys and 15 girls can sit together in a row such that between any 2 boys at least 2 girls sit?
 9. In how many ways we can get a sum greater than 17 by throwing six distinct dice.
 10. A test has 4 parts. The first three parts carry 10 marks each and the 4th part carries 20 marks. Assuming that marks are not given in fractions, find the number of ways in which a candidate can get 30 marks out of 50.

OBJECTIVE (Multi Choice)

1. The number of ways in which the letters of the word ARTICLE can be rearranged so that the even places are always occupied by consonants is
 a) 576 b) ${}^4C_3 \times (4!)$ c) $2(4!)$ d) none of these
 2. The number of ways in which 6 men can be arranged in a row so that three particular men are consecutive, is
 a) 4P_4 b) ${}^4P_4 \times {}^3P_3$ c) ${}^3P_3 \times {}^3P_3$ d) none of these
 3. The number of 4 digit numbers that can be made with the digits 1, 2, 3, 4 and 5 in which at least two are identical, is
 a) $4^5 - 5!$ b) 505 c) 600 d) none of these
 4. In an examination of 9 papers a candidate has to pass in more papers than the number of papers in which he fails in order to be successful. The number of ways in which he can be unsuccessful is
 a) 255 b) 256 c) 193 d) 319
 5. From 4 gentlemen and 6 ladies a committee of five is to be selected. The number of ways in which the committee can be formed so that gentlemen are in majority is
 a) 66 b) 156 c) 60 d) none of these
 6. The number of natural numbers which are less than $2 \cdot 10^8$ and which can be written by means of the digits 1 and 2 is
 a) 772 b) 870 c) 900 d) 766
 7. The number of ways in which a mixed double game can be arranged amongst 9 married couples if no husband and wife play in the same game is
 a) 756 b) 1512 c) 3024 d) none of these
 8. The value of the expression ${}^{k-1}C_{k-1} + {}^kC_{k-1} + \dots + {}^{n+k-2}C_{k-1}$ is

- a) ${}^{n+k-1}C_{k+1}$ b) ${}^{n+k-1}C_k$ c) ${}^{n+k}C_k$ d) none of these
9. The number of ways of selecting 10 balls out of an unlimited number of white, red, blue and green balls is
a) 270 b) 84 c) 286 d) 86
10. The number of proper divisors of 1800 which are also divisible by 10, is
a) 18 b) 34 c) 27 d) none of these
11. There are 20 persons among whom two are brothers. The numbers of ways in which we can arrange them around a circle so that there is exactly one person between the brother is
a) 19! b) $2 \times 18!$ c) $2! \cdot 17!$ d) none of these
12. Let $A = \{x \text{ be a prime number and } x < 30\}$. The number of different rational numbers whose numerator and denominator belong to A is
a) 90 b) 180 c) 91 d) none of these
13. Let S be the set of all functions from the set A. If $n(A) = k$, then $n(S)$ is
a) $k!$ b) k^k c) $2^k - 1$ d) 2^k
14. Let A be the set of 4-digit numbers $a_1 a_2 a_3 a_4$ where $a_1 > a_2 > a_3 > a_4$, then $n(A)$ is equal to
a) 126 b) 84 c) 210 d) none of these
15. A teacher takes 3 children from her class to the Zoo at a time as often as she can, but she does not take the same three children to the zoo more than once. She finds that she goes to the zoo 84 times more than a particular child goes to the zoo. The number of children in her class is
a) 12 b) 10 c) 60 d) none of these

ASSIGNMENT

SECTION- I

PART- A (Level- I)

- Find the number of ways in which 4 different letters of the word MATHEMATICS can be arranged.
- Find the total number of parallelograms that can be formed using ' n_1 ' parallel lines in one direction and ' n_2 ' parallel lines in another direction.
- How many three digit even numbers can be formed by using the digits 1,2,3,4,5,6 (repetition is not allowed).
- Find the number of ways in which a mixed doubles tennis game can be arranged between 10 players consisting of 6 men and 4 women.
- In a hockey tournament, a total of 153 matches were played. If each team played one match with every other team, find the total number of teams that participated in a tournament.
- (i) If 7 points out of 12 are in the same straight line, then find the number of triangles that can be formed by selecting any 3 of them
(ii) Under the same situation, find the number of straight lines that can be formed by joining these points.
- A committee of 5 persons is to be formed out of 6 gents and 4 ladies. Find the number of ways in which this can be done, when at least two ladies are included.
- Find the number of ways in which the letters of the word 'CARACAS' can be arranged so that the C's do not appear together.
- Find the number of ways of arranging six persons (having A, B, C and D among them) in a row so that A, B, C and D are always in order ABCD (not necessarily together).
- If ${}^m C_2 = n$, then find the value of ${}^{m+1} C_4$.

LEVEL-II

- In how many ways can 7 men and 7 women be seated around a circular table so that no two men/no women sit next to each other?
- Suppose that at a sports dinner we have 16 cricketers and 6 tennis players. In how many ways can we seat them at a long table if (i) none of the tennis players is seated next to another tennis player and (ii) all tennis players are seated together.
- Find the number of ways to give 20 different things to three persons A, B and C so that B gets 2 more than A and C gets 1 more than B.
- In two vans, each has 3 seats in the front and 4 at the back. In how many ways 3 girls and 9 boys can be seated in these vans so that the girls sit together at the back.
- Find the number of divisors of 16200 which are not divisible by 4.
- Find the number of positive integral solutions of $abc = 45$.
- Twelve different letters are to be put in twelve pockets in a row. If five of the pockets are too small for six of the letters than in how many ways can the letters be put in the pockets?
- How many different rectangles are there on a chessboard. How many of these have the area = $3 \times$ area of a small square.
- Seven people go to a cinema and park their cars outside, which are identical. While returning only three of them are able to unlock their cars (assuming that a car can be unlocked only by its own key). Find the total number of ways in which this can be done.
- A question paper consist of two part A and B, Part A has 5 questions with one alternate each and part B has 4 questions. Find the number of ways by which one can attempt the paper when at least one question must be attempted from each part.

11. Find the total number of ways in which a 3- digit number xyz can be formed from the digits 1,2,3,4,5,6,7,8 and 9 such that x,y,z are in A.P. and $x \neq y \neq z$.
12. How many 5 letters word can be formed using the letters of word 'MANAGEMENT' such that if any two alike letters are there then they are always together.
13. Find the largest value of n for which $125!$ is divisible by 6^n .
14. Find the number of numbers of six digits that can be made with the digits 1,2,3,4 if all the digits are to appear in the number atleast once.
15. Two players A and B plays a series of '2n' games. Each game can result in either a win or loss for A. Find the total number of ways in which A can win the series of these games. (All the games are to be played)

PART- B (Multi Choice Questions)

1. A polygon has 44 diagonals. The number of its side is
 a) 9 b) 10 c) 11 d) 12
2. There are n numbered seats around a round table. In how many ways can m($m < n$) persons sit around the round table.
 a) ${}^n C_m \cdot m!$ b) ${}^n C_m \cdot (m-1)!$ c) $\frac{{}^n C_m \cdot (m-1)!}{2}$ d) none of these
3. The number of 10 digit numbers that can be written by using the digit 1 and 2 is
 a) ${}^{10} C_2 + {}^9 C_2$ b) 2^{10} c) $2^{10} - 2$ d) $10!$
4. There are k different books and l copies of each in a college library. The number of ways in which a student can make a selection of one or more books is
 a) $(k+1)^l$ b) $(l+1)^k$ c) $(k+1)^l - 1$ d) $(l+1)^k - 1$
5. Along a railway line there are 20 stations. The number of different tickets required in order so that it may be possible to travel from every station to every station is
 a) 380 b) 225 c) 196 d) 105
6. The number of times of the digits 3 will be written when listing the integer from 1 to 1000 is
 a) 269 b) 300 c) 271 d) 302
7. The number of 4- letter words that can be formed out of the letters of the word "FIITJEE" is
 a) 5040 b) 1260 c) 270 d) 180
8. The number of flags with three strips in order, that can be formed using 2 identical red, 2 identical blue and 2 identical white strips is
 a) 24 b) 20 c) 90 d) 8
9. How many numbers between 5000 and 10,000 can be formed using the digits 1,2,3,4,5,6,7,8,9 when each digit is appearing not more than once in each number?
 a) $5 \times {}^8 P_3$ b) $5 \times {}^8 C_3$ c) $5! \times {}^8 P_3$ d) $5! \times {}^8 C_3$
10. Let T_n denote the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21$, then n is equal to
 a) 5 b) 7 c) 6 d) 4
11. The value of $\sum_{r=1}^{10} r \cdot {}^r P_r$ is
 a) ${}^{11} P_{11}$ b) ${}^{11} P_{11} - 1$ c) ${}^{11} P_{11} + 1$ d) none of these
12. The number of n digit numbers, no two consecutive digits being the same, is
 a) $n!$ b) $9!$ c) 9^n d) 9^n
13. A letter lock consists of three rings marked with 15 different letters. If N denotes the number of ways in which it is possible to make unsuccessful attempts to open the lock then,
 a) N is not divisible by 482 b) N is product of 3 distinct prime numbers
 c) N is product of 4 distinct prime numbers d) none of these
14. The numbers of ways of dividing 15 men and 15 women into 15 couples, each containing a man and a woman, is
 a) 1240 b) 1840 c) 1820 d) 1220
15. In a library there are n different books on mathematics, and one can draw any number of books. If the number of ways in which the books can be drawn is 4095, then
 a) $n=9$ b) $n=10$ c) $n=11$ d) $n=12$.
16. The number of integers between 1 and 1000(both inclusive), which are divisible by either of 10, 15 or 25 is
 a) 206 b) 140 c) 146 d) 854
17. The number of numbers less than 1000 that can be formed out of the digits 0,1,2,3,4 and 5, no digit being repeated, is
 a) 130 b) 131 c) 156 d) none of these
18. The number of ordered triplets of positive integers which are solutions of the equation $x+y+z=100$ is
 a) 5081 b) 6005 c) 4851 d) 4554
19. Number of ways of forming a team consisting of atleast 2 and atmost (n-2) students from n students is
 a) $2^n - 2n - 2$ b) $2^n - 2n - 4$ c) $2^n - 2n$ d) 2^{n-1}
20. The number 3 can be written as 3, 2+1, 1+2 or 1+1+1 in four ways. In how many ways can the number n be written?

- a) $n!$ b) $2(n-1)$ c) $(n-1)^2$ d) 2^{n-1}
21. How many six digit numbers can be formed in decimal system in which every succeeding digit is greater than its preceding digit
- a) 9P_6 b) ${}^{10}P_6$ c) 9C_3 d) none of these
22. Total number of 4 digit numbers such that sum of the digit is odd, is
- a) 2720 b) 5040 c) 3680 d) 4500
23. The numbers of ways of selecting two numbers from the set $\{1,2,3,\dots,12\}$ whose sum is divisible by 3 is
- a) 66 b) 16 c) 6 d) 22
24. There are 24 balls of 6 different size in a bag, there being 4 balls of each size in four different colours. In how many ways 4 balls can be selected so that they are of different colours is
- a) 1296 b) 36 c) 3840 d) none of these

Multiple Choice Question (Multiple Options Correct)

1. If m and n are positive integers more than or equal to 2, $m > n$, then $(mn!)$ is always divisible by
- a) $(m!)^n$ b) $(n!)^m$ c) $(m+n)!$ d) $(m-n)!$
2. The number of numbers between n and n^2 which are divisible by n is $(n \in \mathbb{N})$
- a) Less than n b) $n-1$ c) $n-2$ d) greater than n
3. The number of ways in which we can choose two distinct integers from 1 to 100 such that difference between them is at most 10 is
- a) ${}^{100}C_2 - {}^{90}C_2$ b) 945 c) ${}^{100}C_2 - {}^{90}C_{88}$ d) ${}^{90}C_1 + \sum_{r=1}^9 r$

COMPREHENSION TYPE

Read the following write up carefully and answer the following questions:

P is a set containing n elements. A subset A of P is selected at random. The set P is reconstructed by replacing its elements. A subset B of P is again selected at random.

1. The number of ways of selecting the subsets A and B such $A \cup B$ is equal to P is
- a) 3^n b) 2^n c) $(3^n - 1)/2$ d) $(3^n + 1)/2$
2. The number of ways of selecting the subsets such that $A \cap B$ is equal to ϕ is
- a) 3^n b) 2^n c) $(3^n - 1)/2$ d) $(3^n + 1)/2$
3. The number of unordered pairs (A, B) such that $A \cap B$ is ϕ , is
- a) 3^n b) 2^n c) $(3^n - 1)/2$ d) $(3^n + 1)/2$

NUMERICAL BASED

1. The exponent of 7 in ${}^{100}C_{50}$ is.....?
2. An n - digit number is a positive number with exactly n - digits. Nine hundred distinct n - digit numbers are to be formed using only three digits 2,5 and 7. Then the smallest value of n for which this is possible is.....?

MATCH THE FOLLOWING

Column – I

- a) Number of triangle that can be made using the vertices of a polygon of 10 sides as their vertices and having exactly one side common with the polygon is
- b) Number of triangle that can be made using the vertices of a polygon of 10 sides as their vertices and having exactly 2 sides common with the polygon is
- c) Number of Quadrilaterals that can be made using the vertices of a polygon of 10 sides as their vertices and having exactly 2 sides common with the polygon is
- d) Number of Quadrilaterals that can be made using the vertices of a polygon of 10 sides as their vertices and having exactly 3 sides common with the polygon is

Column – II

- p) 75
- q) 110
- r) 60
- s) 10
- t) 90

SECTION – II (Multi Choice Questions/ Single Option Correct)

1. Every one of the 10 available lamps can be switched on to illuminate certain hall. The total number of ways in which the hall can be illuminated is
- a) 55 b) 1023 c) 2^{10} d) 20!
2. In an examination there are 3 multiple choice questions and each question has 4 choices. Number of sequences in which a student can fail to get all answer correct is
- a) 11 b) 15 c) 80 d) 63
3. How many different signals can be given using any number of flags from 5 flags of different colours is
- a) 325 b) 425 c) 525 d) 625
4. The number of 9 digit numbers that can be formed by using the digits 1,2,3,4 and 5 is
- a) ${}^9C_1 \times {}^8C_2$ b) 5^9 c) 9C_5 d) 9!

5. From a class of 25 students 10 are to be chosen for an excursion party. There are 3 students who decide that either all of them or none of them will join. The number ways they can be chosen is
 a) ${}^{22}C_7 + {}^{22}C_{10}$ b) ${}^{22}C_6 + {}^{22}C_{10}$ c) ${}^{22}C_7 + {}^{22}C_8$ d) none of these
6. Number of all 4- digit numbers having different digits formed of the digits 1,2,3,4,5 and divisible by 4 is
 a) 24 b) 30 c) 125 d) 100
7. The sum of proper divisors of 72(1 and 72 are excluded) is
 a) 195 b) 122 c) 194 d) none of these
8. A male and a female typist are needed in an Institution. If 10 ladies and 15 gentlemen apply, then in how many ways can the selection be made, given that one particular pair does not wish to be together.
 a) 125 b) 145 c) 149 d) none of these
9. The total number of selections of atleast one thing at most r things from $(2r+1)$ different things is 255. Then the value of r is
 a) 3 b) 4 c) 5 d) none of these
10. If $x = \{1,2,3,4,5,6,7\}$ then the number of proper subsets of x, containing 7, is
 a) 60 b) 61 c) 62 d) 63
11. For $2 \leq r \leq n$ ${}^nC_r + 2{}^nC_{r-1} + {}^nC_{r-2}$ is equal to
 a) ${}^{n+1}C_{r-1}$ b) $2{}^{n+1}C_{r-1}$ c) ${}^{n-2}C_r$ d) $2{}^{n+2}C_r$
12. The number of zeroes at the end of $(127)!$ is
 a) 31 b) 30 c) 0 d) 10
13. If ${}^nP_r = 5040 ({}^{n-1}C_{r-1} + {}^{n-1}C_r)$, then
 a) $r=5$ b) $r=6$ c) $r=7$ d) $r=8$
14. Number of ways in which 5 identical objects can be distributed in 8 persons such that no person gets more than one object is
 a) 8 b) 8C_5 c) 8P_5 d) none of these
15. Sum of all divisors of 5400 whose units digit is 0 is
 a) 5400 b) 10800 c) 16800 d) none of these.