THE GURUKUL INSTITUTE

PLOT 5C, 2ND FLOOR, GANAPATI COMPLEX, SEC-13, OPP. JAIPURIA SCHOOL, VASUNDHARA, GHAZIABAD (U.P)

CLASS TEST XII

CONTINUITY & DIFFERENTIABILITY

1. Determine the values of a, b and c for which the function

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x}, & x < 0 \\ c, & x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{b\sqrt{x^3}}, & x > 0 \end{cases}$$

   May be continuous at x = 0 .

2. If the function f(x) = $\begin{cases} 3ax + b, if\ x > 1 \\ 11, if\ x = 1 \\ 5ax - 2b, if\ x < 1 \end{cases}$ is continuous at x – 1, find the values of a and b.

3. If y = $\sqrt{\frac{(x-3)(x^2+5)}{3x^2+4x+5}}$, find $\frac{dy}{dx}$.
4. F(x) = $x^{2/3}$ on [-1, 1].
5. F(x) = cos 2x in $[-\frac{\pi}{4}, \frac{\pi}{4}]$.
6. If x = a sin pt and y = b cos pt, find the value of $\frac{d^2y}{dx^2}$ at t = 0.
7. Prove the following, using properties of determinants: $\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ y+z & z+x & x+y \end{vmatrix}$ = (x - y)(y - z) (z – x) (x +y +z).
8. Evaluate, $\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$.
9. If A = $\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$, B = $\begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$, then verify (AB)' = B'A'.
10. Find the value of x, if : $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = 0$.
11. Find weather the following system of equations is consistent or not, find the solution of the system also 3x – y + 2z = 3, x- 2y -z = 1 , 2x + y + 3z = 5.
12. Using elementary transformations, find the inverse of the following matrix: = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$
13. Differentiate $e^x + ee^{x^2} + e^{x^3} + \cdots + e^{x^5}$.
14. Find dy/dx, if $y^x + x^y + x^x = a^b$.
15. Find all the points of discontinuity of the function f defined by

$$f(x) = \begin{cases} x + 2, if\ x < 1 \\ 0, if\ x = 1 \\ x - 2, if\ x > 1 \end{cases}$$

16. Examine the continuity of the function $f(x) = 2x^3 - 1$ at x = 3.
17. Differentiate $2\sqrt{\cot(x^2)}$ w.r.t x.
18. If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1} and hence prove that $A^2 - 4A - 5I = 0$.