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DETERMINANT

Properties of Determinants

1. Without expanding, show that $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$.

2.Evaluate $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & \omega & \omega \end{vmatrix}$ where ω is cube root of unity.

PROBLEM

1.Show that $\begin{vmatrix} -bc & ca+ab & ca+ab \\ ab+bc & -ca & ab+bc \\ bc+ca & bc+ca & -ab \end{vmatrix} = (ab)^3$

2. If $\begin{vmatrix} x^3+1 & x^2 & x \\ y^3+1 & y^2 & y \\ z^3+1 & z^2 & z \end{vmatrix} = 0$ and x,y,z are all different, then prove that xyz = -1.

3. Without expanding a determinant at any stage, show that

$\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = Ax+B$, where A and B are determinants of order 3 not involving x.

OBJECTIVE

1. If $\begin{vmatrix} x & 3 & 6 \\ 3 & 6 & x \\ 6 & x & 3 \end{vmatrix} = \begin{vmatrix} 2 & x & 7 \\ x & 7 & 2 \\ 7 & 2 & x \end{vmatrix} = \begin{vmatrix} 4 & 5 & x \\ 5 & x & 4 \\ x & 4 & 5 \end{vmatrix} = 0$ then x is equal to

a) 9 b) -9 c) 0 d) none of these

2.If a, b, & c are sides of a ΔABC and

$\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+1)^2 & (b+1)^2 & (c+1)^2 \\ (a-1)^2 & (b-1)^2 & (c-1)^2 \end{vmatrix} = 0$, then

a) ΔABC is an equilateral triangle b) ΔABC is a right angled triangle
c) ΔABC is an Isosceles triangle d) none of these

3. If $\begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix} = k(xyz)$, then k is equal to

a) 4 b) -4 c) zero d) none of these

4. $\begin{vmatrix} 109 & 102 & 95 \\ 6 & 13 & 20 \\ 1 & -6 & -13 \end{vmatrix}$ is equal to

a) constant other than zero b) zero c) 100 d) -1997

5. $\Delta = \begin{vmatrix} 18 & 40 & 89 \\ 40 & 89 & 198 \\ 89 & 198 & 440 \end{vmatrix}$ is equal to

a) 1 b) -1 c) zero d) $\Delta = 2$

EXRCISE – 1

1. Evaluate Δ only by using the properties of determinant where

$$\Delta = \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix}.$$

2. Evaluate $\begin{vmatrix} 2^2 & 3^2 & 4^2 \\ 3^2 & 4^2 & 5^2 \\ 4^2 & 5^2 & 6^2 \end{vmatrix}$

3. Find x when $\begin{vmatrix} x+a & a^2 & a^3 \\ x+b & b^2 & b^3 \\ x+c & c^2 & c^3 \end{vmatrix} = 0$, where a,b,c are distinct numbers ($a\neq b\neq c$).

4.If ω is the complex cube root of unity, then prove that $\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{vmatrix} = \pm 3\sqrt{3}i$.

5.If a,b,c (all positive) are the pth, qth and rth terms respectively of a geometric progression , then

prove that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$.

6.It a,b and c are pth, qth and rth terms of an H.P. then $\begin{vmatrix} bc & ca & ab \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$ is

a) p+q+r b) p+q – r c) 0 d) none of these

7.The value of the determinants $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix}$ is

a) 0 b) 1/a+1/b+1/c c) a+b+c d) none of these

8.The value of $\begin{vmatrix} x & x^2 - xy & 1 \\ y & y^2 - zx & 1 \\ z & z^2 - xy & 1 \end{vmatrix}$ is

a) 1 b) -1 c) 0 d) –xyz

9.The value of determinant $\begin{vmatrix} {}^5C_0 & {}^5C_3 & 14 \\ {}^5C_1 & {}^5C_4 & 1 \\ {}^5C_2 & {}^5C_5 & 1 \end{vmatrix}$ is

a) 0 b) –(6!) c) 80 d) none of these

Product of two Determinants

1.Prove thee following by multiplication of determinants and power co – factor formula.

$$\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix} = \begin{vmatrix} b^2+c^2 & ab & ac \\ ab & c^2+a^2 & bc \\ ac & bc & a^2+b^2 \end{vmatrix} = \begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2.$$

2.Prove that $\begin{vmatrix} 2 & \alpha+\beta+\gamma+\delta & \alpha\beta+\gamma\delta \\ \alpha+\beta+\gamma+\delta & 2(\alpha+\beta)(\gamma+\delta) & \alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta) \\ \alpha\beta+\gamma\delta & \alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta & 2\alpha\beta\gamma\delta \end{vmatrix} = 0.$

3.Let α be a repeated root of the quadratic equation f(x) = 0 A(x), B(x), C(x) be polynomials of degree 3,4 and 5 respectively. Then show that $\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$ is divisible by f(x), where a dash denotes the derivative, with respect to x.

Summation of Determinants

1.Let $\Delta_a = \begin{vmatrix} a-1 & n & 6 \\ (a-1)^2 & 2n^2 & 4n-2 \\ (a-1)^3 & 3n^2 & 3n^2-3n \end{vmatrix}$, show that $\sum_{a=1}^{n} \Delta a = 0$.

5.Let $f(x) = \begin{vmatrix} \sec x & \cos x & \sec^2 x+\cot x \operatorname{cosec} x \\ \text{Cos}^2 x & \cos^2 x & \operatorname{cosec}^2 x \\ 1 & \cos^2 x & \cos^2 \end{vmatrix}$ prove that $\int_0^{\pi/2} f(x)dx = -(\pi/4 + 8/15)$

PROBLEM

1. If $\Delta(r) = \begin{vmatrix} (2r) & x & n(n+1) \\ (6r^2-1) & y & n^2(2n+3) \\ (4r^3-2nr) & z & n^2(n+1) \end{vmatrix}$, where n∈N, then prove that $\sum_{r=1}^{n} \Delta(r) = 0$.

2. For all values of A,B,C, and P,Q,R, show that $\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} = 0$.

OBJEECTIVE

1. If $f(\theta) = \begin{vmatrix} 1 & 1 & -1 \\ 1 & e^{I\theta} & 1 \\ 1 & -1 & -e^{I\theta} \end{vmatrix}$ then

a) $\int_{-\pi/2}^{\pi/2} f(\theta)d\theta = 2\int_0^{\pi/2} f(\theta)d\theta$ b) $f(\theta)$ is purely real
c) $f(\pi/2) = 2$ d) none of these

2. $\Delta = \begin{vmatrix} my+nz & mq-nr & mb+nc \\ kz-mx & kr-mp & kb-ma \\ nx+ky & np+kq & na+kb \end{vmatrix}$ is equal to

a) $\Delta = 0$ b) $\Delta \neq 0$ c) $\Delta = f(x,y,z)$ d) none of these

EXERCISE

1. $D_r = \begin{vmatrix} r & n+1 & 1 \\ r^2 & 2n-1 & (2n+1)/3 \\ r^3 & 3n+2 & n(n+1)/2 \end{vmatrix}$, show that $\sum_{r=1}^{n} Dr = 0$.

2. If p(x), q(x) and r(x) are three polynomials of degree 2, then prove that
$\begin{vmatrix} p(x) & q(x) & r(x) \\ p'(x) & q'(x) & r'(x) \\ p''(x) & q''(x) & r''(x) \end{vmatrix}$ is independent of x.

3. If $F(\theta) = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$, then show that $F(\alpha)F(\beta) = F(\alpha+\beta)$.

4.If $\Delta(x) = \begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}$, evaluate $\Delta'(x)$.

5.The value of determinant $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$ is

a) $(a-1)^3$ b) $(1-a)^3$ c) 0 d) none of these

6.The value of $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$ is

a) $(a+b+c)^3$ b) $(a+b-c)^3$ c) $(a-b-c)^3$ d) none of these

7.If $D_1 = \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$ and $D_2 = \begin{vmatrix} b+c & a+c & a+b \\ a+b & c+b & c+a \\ c+a & a+b & b+c \end{vmatrix}$, then

a) $D_1 = 2D_2$ b) $D_2=2D_1$ c) $D_1 =D_2$ d) none of these

8.The value of $\begin{vmatrix} b^2 - ab & b - c & bc - ac \\ ab - a^2 & a - b & b^2 - ab \\ bc - ac & c - a & ab - a^2 \end{vmatrix}$ is

a) 0 b) a+b+c c) $(a+b-c)^2$ d)none of these

Homogeneous System

1. For what value of k do the following homogeneous system of equation posses a non – trivial solution: $x+ky+3z=0$, $3x+ky-2z=0$, $2z+3y-4z=0$.

2.For what values of p and q, the system of equation $2x+py+6z=8$, $x+2y+qz=5$, $x+y+3z=4$ has (i) no solution (ii) a unique solution (iii) infinitely many solutions.

PROBLEM

1. Find the value of 'λ' for which the set of equations
$x+y-2z=0$, $2x-3y+z=0$, $x-5y+4z=\lambda$ are consistant.

2.Let λ and α be real. Find the set of all values of λ for which the system of linear equations
$\lambda x+\sin\alpha.\,y+\cos\alpha.z=0$, $x+\cos\alpha.y+\sin\alpha.z=0$, $-x+\sin\alpha.y-\cos\alpha.z=0$ has a non – trivial solution. For $\lambda=1$, find all values of α.

OBJECTIVE

1.The system of equations $x+2y+3z=4$, $2x+3y+4z=5$, $3x+4y+5z=6$ has

a) infinitely many solutions b) no solutions
c) a unique solution d) none of these

2.If the system of equation $2x+5y+8z=0$, $x+4y+7z=0$, $6x+9y-\lambda z=0$ has a non – trivial solution, then λ is equal to

a) 12 b) -12 c) 0 d) none of these

EXERCISE – 3

1.Find all values of k for which the following system has a non trivial solution

$x+ky+3z=0$
$kx+2y+2z=0$
$2x+3y+4z=0$

2.Find the value of λ and μ for which the following system of equations

$x+y+z=6$
$x+2y+3z=14$
$2x+5y+\lambda z=\mu$ where $\lambda,\mu \in R$ have

a) unique solution, b) infinitely many solution.

3.If the system of linear equations $a(y+z)-x=0$, $b(z+x)-y=0$, $c(x+y)-z=0$ has a non trivial solution, ($a\neq -1$, $b\neq -1$, $c\neq -1$), then show that $\frac{1}{1+a}+\frac{1}{1+b}+\frac{1}{1+c}=2$

4.If the planes $x= cy+bz$, $y=az+cx$, $z=bx+ay$ have a common point other than the origin.
Prove that $a^2+b^2+c^2+2abc=1$.

5.The system of equations

$2x-y+z=0$
$x-2y+z=0$
$\lambda x-y+2z=0$

has infinite number of nontrivial solution for

a) $\lambda=1$ b) $\lambda=5$ c) $\lambda = -5$ d) none of these

6.The system of equation $2x+3y=8$, $7x - 5y+3=0$, $4x - 6y+ \lambda=0$ is solvable if λ is

a) 6 b) 8 c) -8 d) -6

7.The system of equation $ax+4y+z=0$, $bx+3y+z=0$, $cx+2y+z=0$ ha non trivial solution if a,b,c are in

a)A.P. b) G.P. c) H.P. d) none of these

8.The equation $x+y+z=6$, $x+2y+3z=10$, $x+2y+mz=n$ give infinite number of value of the triplet (x,y,z) if

a) $m=3, n\in R$ b) $m=3, n\neq 10$ c) $m=3, n=10$ d) none of these

MISCELLANEOUS PROBLEM

1.For what value of m does the system of equation $3x+my=m$ and $2x - 5y=20$ has a solution satisfying the conditions $x>0, y>0$.

2.If $S_i = a^i+b^i+c^i$, then prove that $\begin{vmatrix} S_0 & S_1 & S_2 \\ S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \end{vmatrix} = (a-b)^2 (b-c)^2 (c-a)^2$.

OBJECTIVE

1.Let $ax^7 + bx^6 + cx^5 +dx^4 +ex^3 + fx^2 + gx+h = \begin{vmatrix} (x+1) & (x^2+2) & (x^2+x) \\ (x^2+x) & (x+1) & (x^2+2) \\ (x^2+2) & (x^2+x) & (x+1) \end{vmatrix}$. Then

a) $g = 3$ and $h = -5$ b) $g= -3$ and $h =-5$ c) $g= -3$ and $h =-9$ d) none of these

2. $\Delta = \begin{vmatrix} p & 2-i & i+1 \\ 2+i & q & 3+i \\ 1-i & 3-i & r \end{vmatrix}$ is always

a) real b) imaginary c) zero d) none of these

3.If $\Delta = \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$. then

a) $\Delta = (a-b)(b-c)(c-a)$ b) a,b,c are in G.P. c) b,c,a are in G.P. d) a,c,b are in G.P.

4.If $\Delta = \begin{vmatrix} x^n & x^{n+2} & x^{2n} \\ 1 & x^p & p \\ x^{n+5} & x^{p+6} & x^{2n+5} \end{vmatrix} = 0$, then p is equal to

a) x^n b) $(n+1)$ c) either A or B d) both A and B

5. $\Delta = \begin{vmatrix} 0 & i-100 & i-500 \\ 100-i & 0 & 1000-i \\ 500-i & i-1000 & 0 \end{vmatrix}$ is equal to

a) 100 b) 500 c) 1000 d) 0

6. $\Delta = \begin{vmatrix} 0 & p-q & a-b \\ q-p & 0 & x-y \\ b-a & y-x & 0 \end{vmatrix}$ is equal to

a) 0 b) a+b c) x+y d) p+q

ASSIGNMENT
SECTION – I
PART – A

1. $\Delta_r = \begin{vmatrix} x & y & z \\ 2^r & 2\times3^r & 3\times4^r \\ 2(2^n-1) & 3(3^n-1) & 4(4^n-1) \end{vmatrix}$ .Find the value of $\sum_{r=1}^{n} \Delta r$.

2.If $\Delta = \begin{vmatrix} a & b & c \\ 6 & 4 & 3 \\ X & x^2 & x^3 \end{vmatrix}$, find $\int_0^1 \Delta dx$.

3.Let $f(x) = \begin{vmatrix} cox & x & 1 \\ 2sinx & x^2 & 2x \\ tanx & x & 1 \end{vmatrix}$, evaluate lim f'(x)/x.

4. Show that $\begin{vmatrix} {}^xC_r & {}^xC_{r+1} & {}^xC_{r+2} \\ {}^yC_r & {}^yC_{r+1} & {}^yC_{r+2} \\ {}^zC_r & {}^zC_{r+1} & {}^zC_{r+2} \end{vmatrix} = \begin{vmatrix} {}^xC_r & {}^{x+1}C_{r+1} & {}^{x+2}C_{r+2} \\ {}^yC_r & {}^{y+1}C_{r+1} & {}^{y+2}C_{r+2} \\ {}^zC_r & {}^{z+1}C_{r+1} & {}^{z+2}C_{r+2} \end{vmatrix}$

5.If $f(\theta) = \begin{vmatrix} \cos^2\theta & \cos\theta\sin\theta & -\sin\theta \\ \text{Cos}\theta\sin\theta & \sin^2\theta & \cos\theta \\ \sin\theta & -\cos\theta & 0 \end{vmatrix}$, then prove that f(θ) is an identity.

6.For what value of k do the following system of equations possess a non-trivial solution over the set of rationals : 3x+ky – 2z=0, x+ky+3z=0,2x+3y – 4z=0. For that value of k, find all the solutions of the system.

7.Show that the system (sin 3θ)x – y+z=0, (cos 2θ)x+4y+3z=0, 2x+7y+7z=0 has a non-trivial solution if θ =nπ or θ=nπ+$(-1)^n$ π/6, (n integer).

8.Let α_1 ,α_2 and β_1, β_2 be the roots of $ax^2+bx+c=0$ and $px^2+qx+r=0$ respectively. If the system of equations $\alpha_1 y+ \alpha_2 z=0$, $\beta_1 y+ \beta_2 z=0$ has a non – trivial solution, then prove that $b^2/q^2 = ac/pr$.

9.Show that the system 3x – y+4z=3, x+2y – 3z= -2, 6x+5y+αz = -3 has at least one solution for any real α. Find the set of solutions if α = -5.

10.Let $f(x) = \begin{vmatrix} cosx & sinx & cosx \\ cos2x & sin2x & 2cos2x \\ cos3x & sin3x & 3cos3x \end{vmatrix}$, find f' (π/2).

11.prove that $\Delta = \begin{vmatrix} \sin 2A & \sin C & \sin B \\ \sin C & \sin 2B & \sin A \\ \sin B & \sin A & \sin 2C \end{vmatrix} = 0$, where A,B,C are the angles of a triangle.

12.Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \tan x & x & 1 \end{vmatrix}$. Find Lt f'(x)/x

13.If a≠p, b≠q, c≠r and $\begin{vmatrix} p & b & c \\ a & q & c \\ a & b & r \end{vmatrix} = 0$, then find value of p/(p – a)+q/(q – b)+r/(r – c).

14.If $(x_1 - x_2)^2+(y_1 - y_2)^2 = a^2, (x_2 - x_3)^2+(y_2 - y_3)^2=b^2, (x_3 - x_1)^2+(y_3 - y_1)^2=c^2$. Then prove that

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = (a+b+c)(b+c-a)(a+b-c).$$

15.If $ax_1^2 +by_1^2+cz_1^2 = ax_2^2 + by_2^2+ cz_2^2 = ax_3^2 +by_3^2 + cz_3^2 = d$.
$ax_2x_3 + by_2y_3+ cz_2z_3 = ax_3x_1 + by_3y_1 +cz_3z_1 = ax_1x_2 +by_1y_2+cz_1z_2 = f$,

then prove that $\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = (d-f)\,\frac{d+2f}{abc}$

16.If α, β be the roots of the equation $ax^2 + bx+c=0$ and $s_n = \alpha^n + \beta^n$, evaluate

$$\begin{vmatrix} 3 & 1+s_1 & 1+s_2 \\ 1+s_1 & 1+s_2 & 1+s_3 \\ 1+s_2 & 1+s_3 & 1+s_4 \end{vmatrix}$$

PART – B
(Multi Choice Single Correct)

1. If A,B,C are angle of a triangle ABC, then the value of the determinant

$$\begin{vmatrix} \sin A/2 & \sin B/2 & \sin C/2 \\ \sin(A+B+C) & \sin B/2 & \cos A/2 \\ \cos(A+B+C)/2 & \tan(A+B+C) & \sin C/2 \end{vmatrix}$$ is less than or equal to

a) 1/ 2 b)1/4 c) 1/8 d) none of these

2.If $f(x) = \begin{vmatrix} x & \cos x & e^{x^2} \\ \sin x & x^2 & \sec x \\ \tan x & 1 & 2 \end{vmatrix}$ then the value of $\int_{-\pi/2}^{\pi/2} f(x)\,dx$ is equal to

a) 0 b) 1 c) d) none of these

3. Let m be a positive integer and

$\Delta_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \end{vmatrix}$ Then value of $\sum_{r=0}^{m} \Delta r$ I given by

$\sin^2(m^2) \quad \sin^2(m) \quad \sin(m^2)$

a) 0 b) $m^2 - 1$ c) 2^m d) $2^m \sin^2(2^m)$

4.The determinant $\Delta = \begin{vmatrix} a^2(1+x) & ab & ac \\ ab & b^2(1+x) & bc \\ ac & bc & c^2(1+x) \end{vmatrix}$ is divisible by

a) 1+x b) $(1+x)^2$ c) x^2 d) none of these

5. If a, b,c are even natural numbers, than $\Delta = \begin{vmatrix} a-1 & a & a+1 \\ b-1 & b & b+1 \\ c-1 & c & c+1 \end{vmatrix}$ is equal to

a) a+b+c b) $a^2+b^2+c^2$ c) abc d) none of these

6.If α and β are the root of x^2 – p+q=0, then the value of $\begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos\alpha \\ \cos(\alpha-\beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$ is

a) 0 b) 1 c) 2 d) none of these

7.If f(x) = $\begin{vmatrix} \cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x \end{vmatrix}$, then $\int_0^{\pi/2} f(x)dx$ is equal to

a) 1/4 b) -1/3 c) 1/2 d) 1

8.The determinant $\Delta = \begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix}$ is equal to zero if

a) a,b,c are in A.P. b) a,b,c are in G.P. c) a,b,c are in H.P.
d) none of these

9.Let f(x) = $\begin{vmatrix} 1+\sin^2x & \cos^2x & 4\sin 2x \\ \sin^2x & 1+\cos^2x & 4\sin 2x \\ \sin^2x & \cos^2x & 1+4\sin 2x \end{vmatrix}$ Then the maximum value of f(x) is

a) 1 b) 6 c) 4 d) none of these

10. If $\Delta = \begin{vmatrix} 85 & 79 & 73 \\ 2 & 8 & 14 \\ 6 & 12 & 18 \end{vmatrix}$, then

a) $\Delta > 0$ b) $\Delta = 0$ c) $\Delta < 0$ d) none of these

11.If the system of equation x+y+2z=0 , 2x – 3y+z=0, x – 5y+4z= λ has a non – trivial solution, then

a) $\lambda > 0$ b) $\lambda < 0$ c) $\lambda = 0$ d) none of these

12. If $f(x) = \begin{vmatrix} 1 & x & x+1 \\ 2x & x(x-1) & x(x+1) \\ 3x(x-1) & x(x-1)(x-2) & x(x+1)(x-1) \end{vmatrix}$, then f(10) is equal to

a) 1 b) 0 c) 10 d) 100

13. If $\Delta = \begin{vmatrix} 2\cos^2 x & \sin 2x & -\sin x \\ \sin 2x & 2\sin^2 x & \cos x \\ \sin x & -\cos x & 0 \end{vmatrix}$, then $\int_0^{\pi/2}(\Delta + \Delta')dx$ is equal to

a) π b) 2π c) – π d) none of these

14. $\begin{vmatrix} 1 & a & a^2+bc \\ 1 & b & b^2+ac \\ 1 & c & c^2+ab \end{vmatrix}$ is equal to

a) 0 b) 2abc c) 2(a – b)(b – c)(c – a) d) none of these

15. If $\begin{vmatrix} 4x & 6x+2 & 8x+1 \\ 6x+2 & 9x+3 & 12x \\ 8x+1 & 12x & 16x+2 \end{vmatrix} = 0$, then x is equal to

a) x = 0 b) x = -11 c) x = 97 d) x = -11/97

16. $\begin{vmatrix} 103 & 115 & 114 \\ 111 & 108 & 106 \\ 104 & 113 & 116 \end{vmatrix} + \begin{vmatrix} 113 & 116 & 104 \\ 108 & 106 & 111 \\ 115 & 114 & 103 \end{vmatrix}$ is equal to

a) a positive number b) a negative number c) zero
d) none of these

17. $\Delta = \begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix}$ is equal to

a) abc b) $a^2b^2c^2$ c) ab+bc+ca d) none of these

18. $\Delta = \begin{vmatrix} x^2+y^2 & ax+by & px+qy \\ ax+by & a^2+b^2 & ap+bq \\ px+qy & ap+bq & p^2+q^2 \end{vmatrix}$ is equal to

a) p+q b) a+b+c c) x+y+z d) 0

19. $\Delta = \begin{vmatrix} 1+a^2+a^4 & 1+ab+a^2b^2 & 1+ac+a^2c^2 \\ 1+ab+a^2b^2 & 1+b^2+b^4 & 1+bc+b^2c^2 \\ 1+ac+a^2c^2 & 1+bc+b^2c^2 & 1+c^2+c^4 \end{vmatrix}$ is equal to

a) $(a+b+c)^6$ b) $(a-b)^2(b-c)^2(c-a)^2$ c) 4(a – b)(b – c)(c – a)
d) none of these

Multi Choice Multi Correct

1. Let $\Delta(x) = \begin{vmatrix} \cos 2x & \sin^2 x & \cos 4x \\ \sin^2 x & \cos 2x & \cos^2 x \\ \cos 4x & \cos^2 x & \cos 2x \end{vmatrix} = a_0 + a_1 \sin x + a_2 \sin^2 x + \ldots\ldots$, then

a) $a_0 = 0$ b) $a_1 = 0$ c) $a_2 = 18$ d) $a_1 = 5$

2. If $\Delta(x) = \begin{vmatrix} x+a & x+b & x+c \\ x+b & x+c & x-1 \\ x+a-c & x-1 & x-b+d \end{vmatrix}$ and $\int_0^2 \Delta(x)dx = -16$, where a,b,c,d are terms of an A.P., then common difference of this A.P. equation could be

a) 2 b) -2 c) -2, 5 d) none of these

NUMERICAL BASED

1.If x,y,z are different from 0 and $\Delta = \begin{vmatrix} a & b-y & c-z \\ a-x & b & c-z \\ a-x & b-y & c \end{vmatrix} = 0$, then value of the expression $a/x + b/y + c/z$ is?

2. If $\begin{vmatrix} x^2+1 & 1 & x+1 \\ 2x^2-1 & 1 & x+2 \\ 3x^2-2 & 1 & x+3 \end{vmatrix} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots\ldots + $ then a_n is equal to?

LINKED COMPREHENSION TYPE

Read the following write up carefully and answer the following questions:

If we expand the determinant $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, then D is a sum of 6 terms which are of the type $a_i b_j c_k$ ($i \neq j \neq k$), i.e., 3!. Half of it will be positive and other half will be negative.

$D = (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2) - (a_1b_3c_2 + a_3b_2c_1 + a_2b_1c_3)$.

1. The number of terms of the type $a_i b_j c_k d_l$ ($i \neq j \neq k \neq l$) in 4×4 determinant must be

a) 12 b) 18 c) 24 d) 30

2.If A . Adj A = $|A| I_n$ where A be a square matrix of order n×n, then |adj A| is

a) $|A|$ b) $|A|^2$ c) $|A|^n$ d) $|A|^{n-1}$

3.Given a n×n matrix A with real entries such that $A^2 = -I$, then |A| must be

a) -1 b) 1 c) 0 d) none of these

Matrix – Match Type

1. Let $f(x) = \begin{vmatrix} \sec^2 x & 1 & 1 \\ \cos^2 x & \cos^2 x & \csc^2 x \\ 1 & \cos^2 x & \cot^2 x \end{vmatrix}$

Column – I	Column – II
a) Period of f(x)	p) $3\pi/32$
b) maximum value of f(x)	q) π
c) $\int_0^{\pi/4} f(x)dx$ - ¼	r) 1

d) minimum value of f(x) s) 0

t) 2

SECTION – II

(Multi Choice Single Correct)

1. $\Delta = \begin{vmatrix} 202 & 197 & 192 \\ 11 & 8 & 5 \\ 107 & 99 & 91 \end{vmatrix}$ is equal to

a) 0 b) 200 c) 100 d) a constant other then 0

2.If $\Delta = \begin{vmatrix} x & 3 & 2 \\ 3 & 2 & x \\ 2 & x & 3 \end{vmatrix} = \begin{vmatrix} 4 & x & 1 \\ x & 1 & 4 \\ 1 & 4 & x \end{vmatrix} = \begin{vmatrix} 100 & -95 & x \\ -95 & x & 100 \\ x & 100 & 95 \end{vmatrix} = \begin{vmatrix} 1002 & -997 & x \\ x & 1002 & -997 \\ -997 & x & 1002 \end{vmatrix} = 0.$

Then x is equal to

a) x =5 b) x = -5 c) x=0 d) none of these

3. If $l_r^2 + m_r^2 + n_r^2 = 1$ where r = 1,2,3 and $l_1l_2 + m_1m_2 + n_1n_2 = 0$…….etc. Then ,

$\Delta = \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$ is equal to

a) 0 b) 2 c) 1 d) 3

4. $\begin{vmatrix} a_1+b_1x & a_1x+b_1 & c_1 \\ a_2+b_2x & a_2x+b_2 & c_2 \\ a_3+b_3x & a_3x+b_3 & c_3 \end{vmatrix} = 0$, then

a) x= 0 b) x =2 c) x= ± 1 d) none of these

5. If the system of equations x – ky – z=0, kx – y – z=0, x+y – z=0 has a non – zero solutions, then possible values of k are

a) -1,2 b) -1,1 c) 1,2 d) 0,1

6.If $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, then f(2x) – f(x) is equal to

a) -1 b) 1 c) 0 d) none of these

7. If $\Delta = \begin{vmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{vmatrix} = 0$, then the non zero root of the equation is

a) –a+b+c b) –a-b-c c) a+b – c d) none of these

8. If $\Delta = \begin{vmatrix} x-3 & 2x+1 & 2 \\ 3x+2 & x+2 & 1 \\ 5x+1 & 5x+4 & 5 \end{vmatrix}$, then Δ is

a) -15 b) 16 c) 15 d) none of these