



# THE GURUKUL INSTITUTE

PLOT 5C, 2ND FLOOR, GANAPATI COMPLEX, SEC-13, OPP. JAIPURIA SCHOOL,  
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TEST – XII<sup>th</sup>

## DIFFERENTIATION

1. Differentiate  $\log \sin (x^x \cdot (a^x)^x)$  w.r.t.  $x$
2. If  $x^y = e^{x-y}$ , then prove that  $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$
3. Differentiate  $\sin^2 \sqrt{x + \log \tan x}$
4.  $\log(\sin \sqrt{x} + e^{\sin \sqrt{x+6}})$
5. Show that  $f(x) = |x| + |x-1|$  is continuous at  $x=0$
6. Differentiate  $(x^x)^x$  w.r.t.  $x$
7.  $x^p y^q = (x+y)^{p+q}$ , prove that  $\frac{dy}{dx} = y/x$
8.  $y = x + \tan x$  prove that  $\cos^2 x \frac{d^2 y}{dx^2} - 2y + 2x = 0$
9. Find  $dy/dx$  when  $y = n^{\log(x \sin x + \log \tan \sin x)}$
10. Differentiate  $(\sin x)^{\log x} + (\tan x)^{x^h}$
11. Find  $dy/dx$ , when  $y = \sin^{-1} (x\sqrt{1-x} + \sqrt{x} \sqrt{1-x^2})$
12. If  $\cos y = x \cos (a+y)$ , then prove  $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$
13. Find  $dy/dx$  when  $y = \sqrt{\sin x + \sqrt{e + \sqrt{\log x}}}$
14. Differentiate  $\frac{(x+1)^3 \sqrt{x^3+3}}{(\sqrt{x+3})^3 \sin^2 \sqrt{x}}$
15. If  $y = \left[ \tan^{-1} \frac{4x}{1+5x^2} \right] + \left[ \tan^{-1} \frac{2+3x}{3-2x} \right]$  then prove that  $\frac{dy}{dx} = \frac{5}{1+25x^2}$
16. If  $x\sqrt{1+y} + y\sqrt{1+x} = 0$  then show that  $\frac{dy}{dx} = -1/(1+x)^2$
17. Find  $dy/dx$  if  $y = \left\{ \log_{\cos x}(\sin x) \right\} - \left\{ \log_{\sin x}(\cos x) \right\} + \sin^{-1} 2x / (1+x^2)$

18. If  $y = \sin \left[ 2 \tan^{-1} \frac{\sqrt{1-x}}{\sqrt{1+x}} \right]$  find  $dy/dx$ .

19. Find  $\frac{d^2y}{dx^2}$  if  $x = a(1-\cos q)$ ,  $y = a(q + \sin q)$

20. Find  $dy/dx$  when  $x = e^{\tan^{-1} \frac{(y-x^2)}{x^2}}$

21. If  $y = 2 \tan^{-1} (\operatorname{cosec} \tan^{-1}(x) - \tan \cot^{-1}(x))$ . Show that  $y = 1/(1+x^2)$

22.  $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$  then prove that  $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

23. If  $y = [x + \sqrt{(x^2+1)}]^h$  then prove  $(1+x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = x^2 y$

24.  $y = x + \frac{1}{x + \frac{1}{x + \dots \infty}}$   $dy/dx (x^2 - y^2 + 3) = 1$

25. If  $y = \frac{2}{\sqrt{a^2 - b^2}} \tan^{-1} \left\{ \frac{\sqrt{(a-b)} \tan x/2}{\sqrt{(a+b)}} \right\}$  show that  $dy/dx = \frac{1}{a + b \cos x}$

26.  $y = \cos^{-1} \left[ \frac{a + b \cos x}{b + a \cos x} \right]$  prove  $dy/dx = \frac{\sqrt{b^2 - a^2}}{b + a \cos x}$

27.  $y = \tan^{-1} \left[ \frac{1}{x^2 + x + 1} \right] + \tan^{-1} \left[ \frac{1}{x^2 + 3x + 3} \right] + \tan^{-1} \left[ \frac{1}{x^2 + 5x + 7} \right]$  find  $dy/dx$ .

28. (a)  $\lim_{x \rightarrow \pi/6} \frac{\sin(x - \pi/6)}{\sqrt{3} - \cos x} \rightarrow 0$  (b)  $\lim_{x \rightarrow 0} \frac{\sqrt{2 - \sqrt{1 + \cos x}}}{\sin^2 x}$

29.  $y = (a^x)^{a^{\dots \infty}}$  prove that  $dy/dx = \frac{y^2 \log y}{x(1 - y \log y)}$

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