



THE GURUKUL INSTITUTE

PLOT 5C, 2ND FLOOR, GANAPATI COMPLEX, SEC-13, OPP. JAIPURIA

SCHOOL, VASUNDHARA, GHAZIABAD (U.P)

MATHEMATICS TEST

CLASS XII

SECTION A

1. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & K & 1 \end{bmatrix}$, Find k , If Cofactor of a_{11} is twice the cofactor of a_{23}
2. Check the monotonicity i.e increasing & decreasing of $f(x) = \cos 2x, [\pi/2, \pi]$.
3. Let $\vec{a} = 5\vec{i} - \vec{j} + 7\vec{k}$, $\vec{b} = \vec{i} - \vec{j} + \lambda\vec{k}$ Find λ such that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.
4. Find $(\vec{i} \times \vec{j}) \cdot \vec{k} + (\vec{k} \times \vec{j}) \cdot \vec{i} - (\vec{i} \times \vec{k}) \cdot \vec{j}$
5. Find the total number of one one function from set A to A if $A = \{1, 2, 3, 4\}$.
6. If $|\vec{a} \times \vec{b}| = 4$, $|\vec{a} \cdot \vec{b}| = 2$, then find $|\vec{a}|^2 |\vec{b}|^2$.
7. Find the angle made by the vector $\vec{i} - 4\vec{j} + 8\vec{k}$ with the z - axis.
8. Given $P(A) = 1/4$, $P(B) = 1/3$ and $P(A \cap B) = 2/3$. Are the events A and B independent?
9. If $|A| = 3$ find the $|A^{-1}|$.
10. find $\int_{-\pi}^{\pi} (\sin^{-93} x + x^{295}) dx$.

TM

SECTION B

11. Let $A = \{-1, 0, 1, 2\}$, $B = \{-4, -2, 0, 2\}$ and $f, g : A \rightarrow B$ be functions defined by $f(x) = x^2 - x$, $x \in A$ and $g(x) = 2 \left| x - \frac{1}{2} \right| - 1$, $x \in A$ are f and g equal. Justify your answer.

12. Prove that the curves $y^2 = 4ax$ and $xy = c^2$ cut at right angles if $c^4 = 32a^4$.

OR

Find the equation of tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$.

13. Prove that $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2 + b^2 + c^2)$.

14. Evaluate : $\int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$.

15. Show that the function $f(x) = \begin{cases} \frac{e^x - 1}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ is discontinuous at $x=0$

16. Form the differential equation of the family of circles having radii 3 .

OR

Solve the differential equation $\frac{dy}{dx} - 3y \cot x = \sin 2x$; $y = 2$ when $x = \frac{\pi}{2}$.

17. Evaluate : $\int_{-1}^1 \{x + [x]\} dx$.

18. A speaks truth in 60% of the cases and B in 70% of the cases. In what percentages of cases they are likely to (i) contradict each other(ii) agree with each other, in stating same fact?

19. If $\vec{a} = \vec{i} + \vec{j} + \vec{k}$, $\vec{c} = \vec{j} - \vec{k}$ are given vectors, find a vector \vec{b} satisfying the equation $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$.

OR

Let $\vec{a} = 2\vec{i} + \vec{k}$, $\vec{b} = \vec{i} + \vec{j} + \vec{k}$ and $\vec{c} = 4\vec{i} - 3\vec{j} + 3\vec{k}$ be three vectors , find a vector \vec{r} which satisfies $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$.

20. Prove that: $4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{70} + \tan^{-1} \frac{1}{99} = \frac{\pi}{4}$.

OR

Solve $\sin^{-1} x + \sin^{-1} (1-x) = \cos^{-1} x$.

21. $y = \log \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right)$, prove that $\frac{dy}{dx} = \frac{x-1}{2x(x+1)}$.

22. Evaluate: $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$

OR

Evaluate: $\int \frac{dx}{(\sin x - 1)(\sin x + 4)}$.

SECTION C

23. Find the inverse of the matrix $\begin{bmatrix} -1 & 2 & 5 \\ 2 & -3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ using elementary column transformation.

24. Suppose the reliability of HIV test is specified as follows. Of people having HIV, 90% of the test detects the disease but 10% go undetected. Of people not having HIV, 99% of the test is judged HIV -ve but 1% are diagnosed as showing HIV + ve. From a large population of which only 0.1% has HIV, one person is selected at random, given the HIV test, and the pathologist reports as HIV +ve. What is the probability that the person actually has HIV?

OR

A fair die is rolled. If 1 turns up, a ball is picked up at random from bag A, if 2 or 3 turns up, a ball is picked up at random from bag B, otherwise a ball is picked up from bag C. Bag A contains 3 red and 2 white balls, bag B contains 3 red and 4 white balls and bag C contains 4 red and 5 white balls. The die is rolled, a bag is picked up and a ball is drawn from it. If the ball is red, what is the probability that bag B was picked up?

25. Define the line of shortest distance between two skew lines .Find the magnitude and the equation of the line of the shortest distance between the following lines : $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$

and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$

26. Using integration, find the area of the region $\{(x, y) : |x-1| \leq y \leq \sqrt{5-x^2}\}$.

27. Kellogg is a new cereal formed of a mixture of barn and rice that contain at least 88 gram of protein and 36 milligram of iron .knowing that barn contain 80 gram of protein and 40 milligram of iron per kg and that rice contain 100 gram of protein and 30milligram of iron per kg, find the minimum cost of producing this new cereal if bran cost ₹ 5 per kg and rice cost ₹ 4 per kg.

28. Find the equation of the plane passing through the point P(1,1,1) and containing the line

$$\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(3\hat{i} - \hat{j} - 5\hat{k}).$$

$$\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(\hat{i} - 2\hat{j} - 5\hat{k}).$$

OR

A variable plane which is at a constant distance p form the origin meets the coordinate axes in points A, B and C respectively. Through these points, planes are drawn parallel to the coordinates planes, show that locus of the point of

intersection is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$.

29. A cylinder of greatest volume is inscribed in a cone, show that (i) $R = \frac{2}{3}h \tan \alpha$ (ii) $H = \frac{1}{3}h$ (iii) Volume of the

cylinder = $\frac{4}{27} \pi h^3 \tan^2 \alpha$. (iv) $r : R = 3 : 2$. Where r, h, α are the radius, height and semi - vertical angle of the cone and r, H are the radius and height of the inscribed cylinder.

