

**PLOT 5C, 2ND FLOOR, GANAPATI COMPLEX, SEC-13, OPP. JAIPURIA
SCHOOL, VASUNDHARA, GHAZIABAD (U.P) PH.No.9810780903
ASSIGNMENT ON RELATION AND FUNCTIONS**

1. If $A = \{1, 2, 3, 4, 5\}$, write the relation $a R b$ such that $a + b = 8$, $a, b \in A$. Write the domain, range & co-domain.
2. Let R be the relation in the set N given by $R = \{(a,b) | a = b - 2, b > 6\}$ Whether the relation is reflexive or not? Justify your answer.
3. Show that the relation R in the set N given by $R = \{(a, b) | a \text{ is divisible by } b, a, b \in N\}$ is reflexive and transitive but not symmetric.
4. Let R be the relation in the set N given by $R = \{(a, b) | a > b\}$
Show that the relation is neither reflexive nor symmetric but transitive.
5. Let R be the relation on R defined as $(a, b) \in R$ iff $1 + ab > 0$, $a, b \in R$.
(a) Show that R is symmetric.
(b) Show that R is reflexive.
(c) Show that R is not transitive.
6. Check whether the relation R is reflexive, symmetric and transitive.
 $R = \{(x, y) | x - 3y = 0\}$ on $A = \{1, 2, 3, \dots, 13, 14\}$.
7. Let $f : R \rightarrow R$ & $g : R \rightarrow R$ be defined as $f(x) = x^2$, $g(x) = 2x - 3$. Find $f \circ g(x)$.
8. If $f : R \rightarrow R$ defined as $f(x) = \frac{2x-7}{4}$ is an invertible function. Find $f^{-1}(x)$.
9. Write the number of all one-one functions on the set $A = \{a, b, c\}$ to itself.
10. Let $*$ be the binary operation on N given by $a * b = \text{LCM of } a \text{ \& } b$. Find $3 * 5$.
11. Let $*$ be the binary on N given by $a * b = \text{HCF of } a, b$, $a, b \in N$. Find $20 * 16$.
12. Let $*$ be a binary operation on the set Q of rational numbers defined as $a * b = \frac{ab}{5}$.
Write the identity of $*$, if any.
13. If a binary operation $*$ on the set of integer Z , is defined by $a * b = a + 3b^2$. Then find the value of $2 * 4$.
14. Show that the function $f : R \rightarrow R$ defined by $f(x) = x^2$ is neither one-one nor onto.
15. Show that the function $f : N \rightarrow N$ given by $f(x) = 2x$ is one-one but not onto.
16. Show that the signum function $f : R \rightarrow R$ given by:

$$F(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$$
is neither one-one nor onto.
17. Let $A = \{-1, 0, 1\}$ and $B = \{0, 1\}$. State whether the function $f : A \rightarrow B$ defined by $f(x) = x^2$ is bijective.
18. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$ are related to each other and all the elements of $\{2, 4\}$ are related to each other. But no element of $\{1, 3, 5\}$ is related to any element of $\{2, 4\}$.
19. Show that each of the relation R in the set $A = \{x \in Z : 0 \leq x \leq 12\}$, given by $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$ is an equivalence relation. Find the set of all elements related to 1.
20. Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2) : T_1 \text{ is similar to } T_2\}$, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related? If R_1 and R_2 are equivalence relations in a set A , show that $R_1 \cap R_2$ is also an equivalence relation.

21. Let $A = \mathbf{R} - \{3\}$ and $B = \mathbf{R} - \{1\}$. Consider the function $f : A \rightarrow B$ defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one-one and onto? Justify your answer.
22. Consider $f : \mathbf{R} \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible and find f^{-1} .
23. On $\mathbf{R} - \{1\}$ a binary operation $*$ is defined as $a * b = a + b - ab$. Prove that $*$ is commutative and associative. Find the identity element for $*$. Also prove that every element of $\mathbf{R} - \{1\}$ is invertible.
24. If $A = \mathbf{Q} \times \mathbf{Q}$ and $*$ be a binary operation defined by $(a, b) * (c, d) = (ac, b + ad)$, for $(a, b) \in (c, d)$ Then with respect to $*$ on A
- examine whether $*$ is commutative & associative:
 - find the identity element in A ,
 - find the invertible elements of A
25. Show that the relation R on A , $A = \{x \mid x \in \mathbf{Z}, 0 \leq x \leq 12\}$, $R = \{(a, b) : |a - b| \text{ is multiple of } 3\}$ is an equivalence relation.
26. Let N be the set of all natural numbers & R be the relation on $N \times N$ defined by $\{(a, b) R (c, d) \text{ iff } a + d = b + c\}$. Show that R is an equivalence relation.
27. Show that the relation R in the set A of all polygons as: $R = \{(P_1, P_2), P_1 \text{ \& } P_2 \text{ have the same number of sides}\}$ is an equivalence relation. What is the set of all elements in A related to the right triangle T with sides 3, 4 & 5?
28. Show that function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by $f(x) = 7 - 2x^3$ for all $x \in \mathbf{R}$ is bijective.
29. Let $A = \mathbf{N} \times \mathbf{N}$ & $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a+c, b+d)$ Show that $*$ is (a) Commutative (b) Associative (c) Find identity for $*$ on A , if any.

