

# IIT/EKLA VYA BATCH

## THE GURUKUL INSTITUTE

PLOT 5C, 2ND FLOOR, GANAPATI COMPLEX, SEC-13, OPP. JAIPURIA  
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### DAILY PRACTICE PROBLEMS

#### DIFFERENTIATION

1. If  $f(x) = \log_{\sec 2x} |\cos 4x| + |\sin x|$  then find  $\frac{dy}{dx}$  at  $x = -\frac{\pi}{6}$  from the first principle.
2. Let  $a+b=1$ ,  $2a^2+2b^2=1$  and  $f(x)$  be a continuous function such that  $f(2+x) + f(x) = 2$  for all  $x \in (0, 2)$  and  $p = \int_0^4 f(x) dx - 4$ ,  $q = \frac{\alpha}{\beta}$  then find the least positive integral values of 'a' for which the equation  $ax^2 - bx + c = 0$  has both roots lies between p and q.
3. Let f be differentiable function such that,  $f^1(x) = f(x) + \int_0^2 f(x) dx$ ,  $f(0) = \frac{4-e^2}{3}$  find  $f(x)$ ?
4. Let  $f(x+y) = f(x) - f(y) + 2xy - 1$  for all  $x, y \in \mathbb{R}$ , if  $f(x)$  is differentiable and  $f^1(0) = \sqrt{-3+a-a^2}$  prove that  $f(x) > 0$  for all  $x \in \mathbb{R}$ .
5. If  $2x = y^{1/5} + y^{-1/5}$  then express y as an explicit function x and prove that  $(x^2 - 1) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 25y$
6. Find  $\frac{dy}{dx}$  if  $y = (\sin x)^{\sin x \sin x \sin x \dots \infty}$
7. Differentiation of a function w.r.t other functions:  $\frac{df(x)}{d\phi(x)} = \frac{\frac{d}{dx}f(x)}{\frac{d}{dx}\phi(x)} = \frac{f^1(x)}{\phi^1(x)}$
8. Differentiate  $\ln \tan x$  with respect to  $\sin^{-1}(e^x)$
9. Differentiate  $\frac{\sin x + \cos x}{\sin x - \cos x}$  with respect to x.
10. If  $\mu = f(x^2)$ ,  $v = g(x^3)$ ,  $f^1(x) = \sin x$  and  $g^1(x) = \cos x$ , then find  $\frac{d\mu}{dv}$ .
11. If  $y^2 = p(x)$  a polynomial of degree 3 then find  $2 \frac{d}{dx} \left( y^3 \frac{d^2y}{dx^2} \right)$  in terms of P(x) and its derivative.
12. If  $x = a(t + \sin t)$  and  $y = a(1 - \cos t)$ , Then find  $\frac{d^2y}{dx^2}$ .
13. If  $y = \sqrt{\frac{1 - \sin 2x}{1 + \sin 2x}}$ , show that  $\frac{dy}{dx} + \sec^2 \left( \frac{\pi}{4} - x \right) = 0$ .
14. Prove that :  $\frac{d}{dx} \left[ \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right] = \sqrt{a^2 - x^2}$ .
15. If  $x = a \left( \frac{1+t^2}{1-t^2} \right)$  and  $y = \frac{2t}{1-t^2}$ , find  $\frac{dy}{dx}$ .
16. If  $f(x) = \left( \frac{3+x}{1+x} \right)^{2+3x}$ , find  $f'(0)$ .
17. If  $x = 3 \sin t - \sin 3t$ ,  $y = 3 \cos t - \cos 3t$ , find  $\frac{d^2y}{dx^2}$  at  $t = \frac{\pi}{3}$ .
18. Differentiate,  $x^x \sin^{-1} \sqrt{x}$  w.r.t x.
19. If  $y = \sqrt{x^2 + 1} - \log \left( \frac{1}{x} + \sqrt{1 + \frac{1}{x^2}} \right)$ , find  $\frac{dy}{dx}$ .
20. Solve the differential equation  $\frac{dy}{dx} - 3y \cot x = \sin 2x$ ;  $y = 2$  when  $x = \frac{\pi}{2}$ .
21. Differentiate  $(x \cos x)^x + (x \sin x)^{\frac{1}{x}}$
22. Solve the differential equation:  $ye^{x/y} dx = \left( xe^{x/y} + y \right) dy$

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