MATHS XII

SECTION A

Q.1 Write the number of all one – one function from the set A with Cartesian number 4 to itself.

Q.2 Write the value of $\tan\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)$

Q.3 For what values of a, $\begin{pmatrix} 2a & -1 \\ -8 & 3 \end{pmatrix}$ is a non singular matrix?

Q.4 Find the value of x, if $\begin{pmatrix} 5 & 3x \\ 2y & z \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 12 & 6 \end{pmatrix}^T$

Q.5 If A is a square matrix of 3 x 3 order and $|A| = 5$, find the value of $|A \text{ adj}A|$

Q.6 Find the value of x for which $x(\hat{i}+\hat{j}+\hat{k})$ is a unit vector.

Q.7 Write a unit vector in XY- plane, making an angle of 30^o with the positive direction of x-axis.

Q.8 Find the distance between two planes: 2x + 3y +4z = 4 and 4x + 6y + 8z = 12.

Q.9 Find value of $\int_0^{\pi/2} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$

Q.10 Evaluate: $\int \frac{\cos 2x}{(\sin x+\cos x)^2}\,dx$

SECTION – B

Q.11 Prove that : $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

OR

Solve the equation: $\tan^{-1}4x = \cot^{-1}\left\{2tan\left(\cos^{-1}\frac{5}{13}\right)\right\} + \tan^{-1}\left\{2tan\left(\sin^{-1}\frac{5}{13}\right)\right\}$

Q.12 Consider the binary operation *** : *R x R →R*** and **O: *R x R →R*** defined **a * b = |a - b|** and **a o b = a** for all a, b ε R. Show that * is commutative but not associative, **O** is associative but not commutative. Further, show that for all a, b, c ε R, **a* (b o c) = (a * b) o (a * c).** Does **O** distributes over *? Justify your answer.

Q.13 Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

Q.14 Prove that $f(x)=|x-1|+|x-2|$ is continuous but not differentiable at x = 2.

OR

Differentiate $(x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

Q.15 Show that the four points (0, -1, -1) , (4, 5, 1) , (3, 9, 4) and (-4, 4, 4) are coplanar. Also find the equation of plane containing them.

Q.16 Solve the differential equation: $ye^{x/y}dx = \left(xe^{x/y} + y\right)dy$

Q.17 Solve the differential equation: $(1 + y + x^2y)\,dx + (x + x^3)\,dy = 0$, $y(1) = 0$

Q.18 Find the interval in which the function $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is strictly increasing or strictly decreasing.

OR

A water tank has the shape of an inverted right cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in tank is 4 m.

Q.19 A die is thrown again and again until three sixes are obtained. Find the probability of obtaining the third six in the sixth throw of the die.

Q.20 Integrate $\int \frac{1}{\sqrt{\sin^3 x \sin(x+\alpha)}}\,dx$

OR

Integrate $\int \frac{1}{\sec x + \sin x}\,dx$

Q.21 If a, b and c are three unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\pi/6$, prove that $\vec{a} = = \pm 2(\vec{b} X \vec{c})$.

Q.22 If $x = \frac{1+\log t}{t^2}$, $y = \frac{3+2\log t}{t}$, $t > 0$ prove that $yy_1 - 2xy_1^2 = 1$

SECTION – C

Q.23 Given that A = $\begin{bmatrix} 1 & 1 & 2 \\ -1 & -2 & 1 \\ 1 & -2 & 3 \end{bmatrix}$ find A^{-1}. Hence using A^{-1} solve the system of equations:

x – y + z = 4, x – 2y – 2z = 9, 2x + y + 3z = 1

OR

Using elementary transformation find the inverse of matrix: $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

Q.24 Using integration find the area of the region enclosed between the circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 1$

Q.25 A manufacturer has three machines I, II, III installed in his factory. Machines I and II are capable of being operated for at most 12 hours where as machine III must be operated for at least 5 hours a day. She produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing I unit of each of M and N on the three machines are given as :

Items	No. of hrs required on machines		
	I	II	III
M	1	2	1
N	2	1	1.25

She makes a profit of Rs 600 and Rs 400 on items M and N resp. How many of each item should she produce so as to maximize her profit assuming that she can sell all the items that she produced? What will be the maximum profit?

Q.26 Find the equation of plane passing through the point (1,1,1) and containing the line $\vec{r} = \left(-3\hat{i} + \hat{j} + 5\hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} + 5\hat{k}\right)$. Also show that the plane contains the line $\vec{r} = \left(-\hat{i} + 2\hat{j} + 5\hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} + 5\hat{k}\right)$

Q.27 Evaluate $\int_{1}^{0} \sin^{-1}\left(x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^{2}}\right)dx$

Q.28 A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{2/3} + b^{2/3})^{3/2}$.

OR

An open toped box is to be constructed by removing equal squares from each corner of a 3 metre by 8 metre rectangular sheet of aluminium and folding up the sides. Find the volume of the largest such box.

Q.29 Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drugs reduces its chance by 25%. At a time a patient can choose any one of two options with equal prob. It is given that after going through one of two options the patient selected at random suffers a heart attack. Find the prob. that the patient followed a course of meditation and yoga.