THE GURUKUL INSTITUTE

PLOT 5C, 2ND FLOOR, GANAPATI COMPLEX, SEC-13, OPP. JAIPURIA SCHOOL, VASUNDHARA, GHAZIABAD (U.P)

Class – XII

MATRICES AND DETERMINANTS

1. How many matrices of order 3 x 3 are possible with each entry as 0 or 1?
2. For any 2 x 2 matrix A, if A (adj A) = $\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then find |A|
3. If A= $\begin{pmatrix} 2 & -3 & -1 \end{pmatrix} and B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} find AB$
4. A matrix A of order 3 x 3 has determinant 7, what is the value of |3A|?
5. If A is a square matrix such that $A^2 = A$, then find $(I + A)^2 - 3A$.
6. If $\begin{bmatrix} 0 & x+2 & 2-x \\ 1-2x & 0 & 2x-1 \\ 3x-8 & x-8 & 0 \end{bmatrix}$ is a skew symmetric, find value of x.
7. If X+Y= $\begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix} and X - Y = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix}$,Find X and Y
8. If A is 3x4 matrix and B is a matrix such that $A^T B$ and BA^T are both defined, find order of mat B.
9. For what value of x if $\begin{vmatrix} -2 & x \\ 3 & -3 \end{vmatrix} = \begin{vmatrix} -x & -1 \\ -6 & -3 \end{vmatrix}$
10. If A is a square matrix of order 3 such that $|adj\ A| = 25$, find $|A|$
11. Show that the matrix A= $\begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$ is Skew-symmetric.

SECTION-B

12. Prove by using properties of determinant $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$
13. If A = $\begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$, then prove that $aA^{-1} = (a^2 + bc + 1)I - aA$
14. If A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B= $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$,Find BA and hence solve equations x-y =3, 2x +3y +4z =17,y+2z =7
15. 16. Using properties of determinants show that $\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$
16. 17. Let A = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, B = $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ Verify that $(AB)^T = B^T A^T$.

17. 18 Using properties of determinants, show that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$ = 2abc(a+b+c) ^{3}

18. If *a, b, c* are in AP, show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix}$ =0.

By using properties of determinants show that: $\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3.$

19. Using properties of determinants, prove that:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

20. Show that A = $\begin{pmatrix} -8 & 5 \\ 2 & 4 \end{pmatrix}$, satisfies the equation $x^2 + 4x - 42 = 0$.Hence find A^{-1}.

21. Using properties of determinants, prove that: $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

22. If a,b,c are in A.P then find the value of the determinant $\begin{vmatrix} x+3 & x+4 & x+5a \\ x+4 & x+5 & x+5b \\ x+5 & x+6 & x+5c \end{vmatrix}$

23. Prove using properties that $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$ = 3(a+b+c)(ab+bc+ca) **.**

24. Show that the function F: R - {-1} →R - {-1}, given by f(x) = $\dfrac{x-3}{x+1}$ is a bijection.Also find f^{-1}

25. If A= $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, then by induction show that $(aI + bA)^n = a^n I + n.a^{n-1}.bA$, if a& b are constants.

26. Express A = $\begin{pmatrix} 2 & -1 & 2 \\ 3 & 4 & 0 \\ 0 & -3 & -2 \end{pmatrix}$ as sum of a symmetric and skew symmetric matrix.

SECTION-C

27. If A = $\begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$ find A^{-1} and hence solve the equations: 2x – 3y + 5z = 11, 3x + 2y – 4z = -5, x + y - 2z = - 3.

28. 32. If A = $\begin{bmatrix} 1 & 1 & 2 \\ -1 & -2 & 1 \\ 1 & -2 & 3 \end{bmatrix}$ find A^{-1}. Hence solve the equations: x – y + z = 4, x – 2y – 2z = 9, 2x + y + 3z = 1

29. Show that $\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (z+y)^2 \end{vmatrix}$ = 2 xyz (x + y + z) ^{3}

30. Using elementary row transformation find the inverse of the matrix $\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}$.

31. Prove using properties that

$$\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xy & yz & (x+y)^2 \end{vmatrix} = 2xyz\,(x+y+z)^3$$

32. Using matrices solve system of equations:x+3y+4z = 8,2x+y+2z =5,5x+y+z =7

33. Let f : N → R be a function defined as f(x) = 4x^2 + 12 x + 15. Show that f : N → S where, S is the range

   a. off , is invertible. Find f $^{-1}$.

35. If A = $\begin{pmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix} and B^{-1} = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix} compute(AB)^{-1}$