



MATRICES AND DETERMINANTS

- How many matrices of order 3×3 are possible with each entry as 0 or 1?
- For any 2×2 matrix A, if $A(\text{adj } A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$, then find $|A|$
- If $A = \begin{pmatrix} 2 & -3 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ find AB
- A matrix A of order 3×3 has determinant 7, what is the value of $|3A|$?
- If A is a square matrix such that $A^2 = A$, then find $(I + A)^2 - 3A$.
- If $\begin{bmatrix} 0 & x+2 & 2-x \\ 1-2x & 0 & 2x-1 \\ 3x-8 & x-8 & 0 \end{bmatrix}$ is a skew symmetric, find value of x.
- If $X+Y = \begin{pmatrix} 5 & 2 \\ 0 & 9 \end{pmatrix}$ and $X-Y = \begin{pmatrix} 3 & 6 \\ 0 & -1 \end{pmatrix}$, Find X and Y
- If A is 3×4 matrix and B is a matrix such that $A^T B$ and BA^T are both defined, find order of mat B.
- For what value of x if $\begin{vmatrix} -2 & x \\ 3 & -3 \end{vmatrix} = \begin{vmatrix} -x & -1 \\ -6 & -3 \end{vmatrix}$
- If A is a square matrix of order 3 such that $|\text{adj } A| = 25$, find $|A|$
- Show that the matrix $A = \begin{pmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{pmatrix}$ is Skew-symmetric.

SECTION-B

- Prove by using properties of determinant $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$
- If $A = \begin{bmatrix} a & b \\ c & \frac{1+bc}{a} \end{bmatrix}$, then prove that $aA^{-1} = (a^2 + bc + 1)I - aA$
- If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, Find BA and hence solve equations $x-y=3$,
 $2x+3y+4z=17$, $y+2z=7$
- Using properties of determinants show that $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ca \\ 1 & c & c^2 - ab \end{vmatrix} = 0$
- Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ Verify that $(AB)^T = B^T A^T$.
- If a, b, c are in AP, show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$.

18. By using properties of determinants show that:
$$\begin{vmatrix} a & a+b & a+b+c \\ 2a & 3a+2b & 4a+3b+2c \\ 3a & 6a+3b & 10a+6b+3c \end{vmatrix} = a^3.$$

19. Using properties of determinants, prove that:

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

20. Show that $A = \begin{pmatrix} -8 & 5 \\ 2 & 4 \end{pmatrix}$, satisfies the equation $x^2 + 4x - 42 = 0$. Hence find A^{-1} .

21. Using properties of determinants, prove that:
$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

22. If a, b, c are in A.P then find the value of the determinant
$$\begin{vmatrix} x+3 & x+4 & x+5a \\ x+4 & x+5 & x+5b \\ x+5 & x+6 & x+5c \end{vmatrix}$$

23. Prove using properties that
$$\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca).$$

24. If $A = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, then by induction show that $(aI + bA)^n = a^n I + n.a^{n-1}.bA$, if a & b are constants.

SECTION-C

25. If $A = \begin{pmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{pmatrix}$ find A^{-1} and hence solve the equations: $2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$,

$$x + y - 2z = -3.$$

26. If $A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & -2 & 1 \\ 1 & -2 & 3 \end{bmatrix}$ find A^{-1} . Hence solve the equations: $x - y + z = 4$, $x - 2y - 2z = 9$,

$$2x + y + 3z = 1$$

27. Using elementary row transformation find the inverse of the matrix
$$\begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}.$$

28. Prove using properties that

$$\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xy & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

29. Using matrices solve system of equations: $x+3y+4z = 8$, $2x+y+2z = 5$, $5x+y+z = 7$.

30. If $A = \begin{pmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{pmatrix}$ and $B^{-1} = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$ compute $(AB)^{-1}$