

MATRICES

Algebra Of Matrices

1. If $A = \begin{pmatrix} 2 & 3 & 1 \\ 1 & 3 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 3 \end{pmatrix}$, show that $AB \neq BA$.

2. If $A_\alpha = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix}$, then find $A_\alpha A_\beta$.

PROBLEMS

1. If a, b, c and d are real numbers and $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,

prove that $A^2 - (a+d)A + (ab - bc)I = O$.

2. Show that the product of two upper or two lower triangular matrices is itself triangular.

OBJECTIVE

1. If A and B are any two square matrices of the same order, then

a) $(AB)' = A'B'$

b) $\text{adj}(AB) = \text{adj}(A)\text{adj}(B)$

c) $(AB)' = B'A'$

d) $AB = O, A = O$ or $B = O$

2. If $A = \begin{pmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 & -1 \\ 2 & 2 & -1 \\ 3 & 0 & -1 \end{pmatrix}$, then

a) $AB = BA$

b) $AB \neq BA$

c) $AB = 1/2 BA$

d) none of these

EXERCISE - 1

Explain why in general (1 - 3):

1. $A^2 - B^2 \neq (A+B)(A-B)$.

2. $(A \pm B)^2 \neq A^2 + B^2 \pm 2AB$.

3. $A^3 \pm B^3 \neq (A \pm B)(A^2 \mp AB + B^2)$.

4. Let $f: S \rightarrow S$ be a function over the set S , where S is a set of all square matrices and $f(x) = x^2 - 5x + 6$,

$\forall x \in S$. Find $f(A)$ if $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$.

5. Prove that the product of the two matrices $\begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{pmatrix}$ and $\begin{pmatrix} \cos^2 \theta & \cos \theta \sin \theta \\ \cos \theta \sin \theta & \sin^2 \theta \end{pmatrix}$

is a null matrix when θ and α differ by an odd multiple of $\pi/2$.

6. If $A = \begin{pmatrix} 0 & -\tan \theta/2 \\ \tan \theta/2 & 0 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, show that $I+A = (I-A) \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

7. Evaluate the following:

a) $\left(\begin{pmatrix} 1 & 3 \\ -1 & -4 \end{pmatrix} + \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} \right) \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$ (b) $\begin{bmatrix} 2 & 3 & 5 \end{bmatrix} \begin{pmatrix} 1 & 0 & 3 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ 6 \end{pmatrix}$

8. Find 'x' if $\begin{pmatrix} x & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} x \\ 3 \end{pmatrix} = 0$

9. Find x such that $\begin{pmatrix} 1 & x & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ x \end{pmatrix} = 0$

10. If $A = \begin{pmatrix} 1 & -2 & 4 \\ 2 & 3 & 2 \\ 3 & 1 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & -1 & 4 \\ 1 & 3 & 2 \\ -1 & 1 & 5 \end{pmatrix}$, then $A+B$ is

a) $\begin{pmatrix} 1 & -2 & 4 \\ 3 & 3 & 2 \\ 2 & 1 & 5 \end{pmatrix}$

b) $\begin{pmatrix} 1 & -2 & 8 \\ 3 & 3 & 4 \\ 2 & 1 & 10 \end{pmatrix}$

c) $\begin{pmatrix} 1 & -4 & 8 \\ 3 & 6 & 4 \\ 2 & 2 & 10 \end{pmatrix}$

d) none of these

11. If $A^2 = 8A + KI$, where $A = \begin{pmatrix} 1 & 0 \\ -1 & 7 \end{pmatrix}$, then value of K is

a) 7

b) -7

c) 1

d) -1

Special Matrices

1. Suppose a, b, c are real numbers such that $abc = 1$. If $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ is such that $A'A = I$, then find the value of $a^3 + b^3 + c^3$.

2. If $\omega \neq 1$ is a cube root of unity, then show that given matrix is a singular matrix.

$$A = \begin{pmatrix} 1 + 2\omega^{100} + \omega^{200} & \omega^2 & 1 \\ 1 & 1 + \omega^{100} + 2\omega^{200} & \omega \\ \omega & \omega^2 & 2 + \omega^{100} + 2\omega^{200} \end{pmatrix}$$

3. Show that the matrix $\begin{pmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{pmatrix}$ is a nilpotent matrix of index 3.

PROBLEM

1. Determine the value of α, β, γ when $\begin{pmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{pmatrix}$ is orthogonal.

2. Show that every square matrix A can be uniquely expressed as $P + iQ$ where P and Q are Hermitian Matrices.

3. If A is Hermitian such that $A^2 = O$, show that $A = O$, where O is the zero matrix.

OBJECTIVE

1. For what value of x, the matrix $\begin{pmatrix} 3-x & 2 & 2 \\ 2 & 4-x & 1 \\ -2 & -4 & -1-x \end{pmatrix}$ is singular.

a) $x=1, 2$

b) $x=0, 2$

c) $x=0, 1$

d) $x=0, 3$.

EXERCISE - 2

1. Show that every square matrix can be uniquely expressed as sum of a symmetric and skew symmetric matrix.

2. If B is a real $m \times n$ matrix, show that $B'B$ as well as BB' is a symmetric matrix.

3. Find the symmetric and skew - symmetric part of the matrix $A = \begin{pmatrix} 1 & 2 & 4 \\ 6 & 8 & 1 \\ 3 & 5 & 7 \end{pmatrix}$.

4. If A, B, A+I and A+B are idempotent matrices show that $AB = BA$.

5. If A and B are two given square matrices then show that

a) $B'AB$ is symmetric if A is symmetric.

b) $B'AB$ is skew - symmetric if A is skew symmetric.

6. Show that the matrix $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.

7. The matrix $\begin{pmatrix} \lambda & 7 & -2 \\ 4 & 1 & 3 \\ 2 & -1 & 2 \end{pmatrix}$ is a singular matrix if λ is

a) 2/5

b) 5/2

c) -5

d) none of these

8. If $f(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then $f(x+y)$ is equal to

a) $f(x)+f(y)$

(b) $f(x) - f(y)$

c) $f(x) \cdot f(y)$

d) none of these

9. If $A = \begin{pmatrix} \alpha & 0 \\ 1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 5 & 1 \end{pmatrix}$ such that $A^2 = B$, then α is

a) 1

b) -1

c) 4

d) none of these

Elementary Transformations or Elementary Operations of a Matrix

1. Using elementary row transformations and find the inverse of the matrix

$$A = \begin{pmatrix} 3 & -1 & -2 \\ 2 & 0 & -1 \\ 3 & -5 & 0 \end{pmatrix}$$

2. If the system of equations $x+ay-z=0$, $2x-y+az=0$ and $ax+y+2z=0$ has a non - trivial solution, then find the value of 'a'.

3. Find the values of 'k' for which the system of equations $(k+1)x + 8y = 4k$, $kx + (k+3)y = 3k - 1$ has no solution.

PROBLEMS

1. If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, show that $A^2 - 4A - 5I = 0$ where I and 0 are the unit matrix and the null matrix of order 3 respectively. Use this result to find A^{-1} .

2. Solve the system of equations $x+3y-2z=0$, $2x-y+4z=0$, $x-11y+14z=0$.

3. Discuss for all values of λ , the system of equation $x+y+4z=6$, $x+2y-2z=6$, $\lambda x+y+z=6$ as regards existence and nature of solutions.

OBJECTIVE

1. The adjoint of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$ is.....

2. If $A = \begin{pmatrix} 2 & 5 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$, then $A^{-1}I$ equal to.....

3. With the help of matrices, the solution of the equations:

$$3x+y+2z=3, \quad 2x-3y-z=-3, \quad x+2y+z=4 \text{ is given by}$$

a) $x=1, y=2, z=-1$

b) $x=-1, y=2, z=1$

c) $x=1, y=-2, z=-1$

d) $x=-1, y=-2, z=1$

4. If $A = \begin{pmatrix} 1 & \tan x \\ -\tan x & 1 \end{pmatrix}$, $x \neq (2n+1)\pi/2$, then the value of $[A^T A^{-1}]$ is

a) $\cos 4x$

b) $\sec^2 x$

c) $-\cos 4x$

d) 1

EXERCISE - 3

Solve the following equation by matrix method (1 - 2)

1. $x+y-2z=1; x-y+z=0; 2x+3y-4z=2,$

2. $x+2y+3z+1=0, 2x+y-z+5=0, 3x+y-2z-3=0.$

3. If A is the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{pmatrix}$ and I is the unit matrix of order 3, then show that $A^3 = pI + qA + rA^2$.

4. Find the adjoint of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{pmatrix}$ and hence, evaluate A^{-1} .

5. For the matrix $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{pmatrix}$, show that $A(\text{adj}A) = 0$.

6. If $A = \begin{pmatrix} 0 & -4 & 1 \\ 2 & \lambda & -3 \\ 1 & 2 & -1 \end{pmatrix}$, then A^{-1} exists (i.e. A is invertible) if

a) $\lambda \neq 4$

b) $\lambda \neq 8$

c) $\lambda = 4$

d) none of these

7. If $A = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 2 & 0 \\ 1 & 3 & 0 \end{pmatrix}$, then the value of $|\text{adj}A|$ is equal to

a) 5

b) 0

c) 1

d) none of these

8. If $A = \begin{pmatrix} \sin \alpha & -\cos \alpha & 0 \\ \cos \alpha & \sin \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then A^{-1} is equal to

a) A^T

b) A

c) $\text{adj}A$

d) none of these

MISCELLANEOUS PROBLEMS

1. Find the possible square roots of the two rowed unit matrix I .

2. $A = \begin{pmatrix} a & 0 & 1 \\ 1 & c & b \\ 1 & d & b \end{pmatrix}$, $B = \begin{pmatrix} a & 1 & 1 \\ 0 & d & c \\ f & g & h \end{pmatrix}$, $U = \begin{pmatrix} f \\ g \\ h \end{pmatrix}$, $V = \begin{pmatrix} a^2 \\ 0 \\ 0 \end{pmatrix}$. If there is vector matrix X , such that $AX = U$ has

infinitely many solutions, then prove that $BX = V$ cannot have a unique solution. If $afd \neq 0$ then prove that $BX = V$ has no solution.

OBJECTIVE

1.If the trace of the matrix : $A = \begin{pmatrix} x-1 & 0 & 2 & 5 \\ 3 & x^2-2 & 4 & 1 \\ -1 & -2 & x-3 & 1 \\ 2 & 0 & 4 & x^2-6 \end{pmatrix}$ is 0 then x is equal to

- a) (-2,3) b) (2, -3) c) (-3,2) d) (3,-2)

2.If A and B are square matrices of order 3, then

- a) $\text{adj}(AB) = \text{adj}A + \text{adj}B$ b) $(A+B)^{-1} = A^{-1} + B^{-1}$
c) $AB = 0 \rightarrow |A| = 0$ or $|B| = 0$ d) $AB = 0 \rightarrow |A| = 0$ and $|B| = 0$

3.The system $AX=B$ of n equations in n unknown has infinitely many solution if

- a) $\det A \neq 0$ b) $\det A = 0, (\text{adj } A)B \neq 0$
c) $\det A = 0, (\text{adj } A)B = 0$ d) $\det A \neq 0, (\text{adj } A)B = 0$

ASSIGNMENT

1.If $D = \text{diag} (a_1, a_2, a_3, \dots, a_n)$, where $a_i \neq 0 \forall i = 1, 2, \dots, n$, then show that

$$D^{-1} = \text{diag} (a_1^{-1}, a_2^{-1}, \dots, a_n^{-1}).$$

2.Find the inverse of the matrix

$$A = \begin{pmatrix} a+ib & c+id \\ -c+id & a-ib \end{pmatrix} \text{ if } a^2+b^2+c^2+d^2=1.$$

3.(i) If A is a non - singular square matrix, then prove that $\text{adj}(\text{adj}(A)) = |A|^{n-2} A$.

(ii) If A is a Hermitian matrix, then prove that $\text{adj}A$ is also Hermitian.

4.If matrix $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ where a,b,c are real numbers and $A^T A = I$, then find the value of a+b+c.

5.Solve the following system of homogeneous equations:

$$2x+3y-z=0; \quad x-y-2z=0; \quad 3x+y+3z=0.$$

6.If A be a square matrix then show that $\text{adj } A' = (\text{adj } A)'$.

7.If A is a diagonal matrix with diagonal elements all different, prove that $AB = BA$ if and only if B is a diagonal matrix.

8. Find the inverse of $\begin{pmatrix} 5 & 2 & -6 \\ 7 & -3 & 4 \\ 3 & 5 & -12 \end{pmatrix}$ by using elementary row operations.

9.Show that the equations $-2x+y+z=a, x-2y+z=b, x+y-2z=c$ have no solutions unless $a+b+c=0$, in which case, they have infinitely many solutions.

10.Investigate for what values of λ, μ the equations

$$x+y+z=6; \quad x+2y+3z=10, \quad x+2y+\lambda z = \mu \text{ have...}$$

- (i) no solutions,
(ii) unique solutions,
(iii) infinitely many solutions.

11. Let $A = [a_{ij}]$, where $a_{ij} u_i v_j \forall 1 \leq i \leq n, 1 \leq j \leq n$ and $u_i, v_j \in \mathcal{R}$ satisfies $A^2 = 16A$, find $\text{tr} (A)$.

12.Show that the matrix $A = \begin{pmatrix} ab & b^2 \\ -a^2 & -ab \end{pmatrix}$ is nilpotent. Also find the index of nilpotency.

13. Show that the system of equations

$$3x - y + 4z = 3; \quad x + 2y - 3z = -2; \quad 6x + 5y + \lambda z = -3$$

Has at least one solution for all real λ . Find the set of solutions if $\lambda = -5$.

14. The square matrix A is an involutory. If a square matrix P is such that $P^2 = P$, then show that $A = 2P - I$ is involutory and $B = 1/2(A+I)$ satisfies the condition $B^2 = B$.

PART - B

(Multi Choice Single Correct)

1. $\begin{pmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix} + 2 \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ is equal to

- a) $\frac{43}{44}$ b) $\frac{43}{45}$ c) $\frac{45}{44}$ d) none of these

2. If $A = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}$, then $|A| |Adj A|$ is equal to

- a) a^{25} b) a^{27} c) a^{81} d) none of these

3. If a matrix A is symmetric as well as skew symmetric then A is a

- a) diagonal matrix b) null matrix c) unit matrix d) none of these

4. If $A = \begin{pmatrix} \alpha & 0 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 16 & 0 \\ 0 & 1 \end{pmatrix}$ then the value of α for which $A^2 = B$ is

- a) 1 b) -1 c) 4 d) none of these

5. If $A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, then

- a) $A^2 = A$ b) $A^2 = 0$ c) $A^2 = I$ d) $A^3 = 0$

6. If $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$, then A is

- a) an invertible matrix b) an idempotent matrix c) a nilpotent matrix
d) none of these

7. If A and B are symmetric matrices of the same order, then

- a) AB is a symmetric matrix b) $A - B$ is skew - symmetric matrix
c) $AB + BA$ is a symmetric matrix d) $AB - BA$ is a symmetric matrix

8. If A is any square matrix, then

- a) $A + A'$ is skew symmetric b) $A - A'$ is symmetric c) AA' is symmetric
d) none of these

9. If A is a square matrix such that $A^3 = I$ then A^{-1} is equal to

- a) I b) A c) A^2 d) none of these

10. If A is any square matrix then which of the following is not symmetric?

- a) $A + A'$ b) $A - A'$ c) AA' d) $A'A$

11. Let A be a skew - symmetric matrix of order n. Then

- a) $|A| = 0$ if n is even b) $|A| = 0$ if n is odd c) $|A| = 0$ for all $n \in \mathbb{N}$

d) none of these

12. Each diagonal element of a skew – symmetric matrix is

- a) zero b) positive c) non – real d) negative

13. If $A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$, then the value of $(A - 2I)(A - 3I)$ is

- a) unit matrix b) Non – singular matrix c) Null matrix d) None of these

14. If $A = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$, $n \in \mathbb{N}$ then A^{4n} is

- a) $\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$ b) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ c) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ d) $\begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}$

15. If $A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$, then A^{-1} is equal to

- a) $\begin{pmatrix} 7 & -3 & -3 \\ 0 & 1 & 0 \\ -1 & 0 & 5 \end{pmatrix}$ b) $\begin{pmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}$ c) $\begin{pmatrix} 7 & -3 & -3 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$

d) none of these

16. If A and B matrices commute then

- a) A^{-1} and B also commute b) B^{-1} and A also commute
c) A^{-1} and B^{-1} also commute d) all the above

17. If A, B, C are three matrices conformable for multiplication then $(ABC)^{-1}$ is equal to

- a) $A^{-1}B^{-1}C^{-1}$ b) $B^{-1}C^{-1}A^{-1}$ c) $C^{-1}B^{-1}A^{-1}$ d) can not said

18. If $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{pmatrix}$, then A^2 is equal to

- a) unit matrix b) null matrix c) A d) -A

19. Trace of a skew symmetric matrix is always

- a) negative b) positive c) zero d) none of these

20. If $A = \begin{pmatrix} \alpha & 2 \\ 2 & \alpha \end{pmatrix}$ and $|A^3| = 125$ then the value of α is

- a) ± 1 b) ± 2 c) ± 3 d) ± 5

(Multi Choice Multi Correct)

1. Let $\alpha = \pi/5$ and $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, then $B = A^4 - A^3 + A^2 - A$ is

- a) singular b) non – singular c) symmetric d) $|B| = 1$

2. If $A = \begin{pmatrix} \alpha & \beta \\ 0 & \alpha \end{pmatrix}$ is the n^{th} root of I_2 , then choose the correct statement

- a) If n is odd, $\alpha = 1, \beta = 0$ b) if n is odd, $\alpha = -1, \beta = 0$
c) If n is even, $\alpha = 1, \beta = 0$ d) If n is even, $\alpha = -1, \beta = 0$

3. If matrix $A = \begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$, where a, b, c are real numbers, $abc = 1$ and $A^T A = I$, then which of the following may be true
- a) $a+b+c=1$ b) $a^2+b^2+c^2=1$ c) $ab+bc+ca=0$ d) $a^3+b^3+c^3=4$

NUMERICAL BASED TYPE

1. If A and B are two non-singular matrices of the same order such that $B^r = I$, for some positive integer $r > 1$. Then $A^{-1}B^{r-1}A - A^{-1}B^{-1}A$ is equal to
2. For any matrix A of order 2×2 , if $A(\text{adj}A) = \begin{pmatrix} 7 & 0 \\ 0 & 7 \end{pmatrix}$, then $|A|$ is equal to?

LINKED COMPREHNSION TYPE

Read the following write up carefully and answer the following questions:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}, \text{ If } U_1, U_2 \text{ and } U_3 \text{ are columns matrices satisfying.}$$

$$AU_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, AU_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}, AU_3 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \text{ and } U \text{ is } 3 \times 3 \text{ matrix whose columns are } U_1, U_2, U_3 \text{ then answer the following questions}$$

1. The value of $|U|$ is
- a) 3 b) -3 c) 3/2 d) 2
2. The sum of the elements of U^{-1} is
- a) -1 b) 0 c) 1 d) 3

3. The value of $\begin{pmatrix} 3 & 2 & 0 \end{pmatrix} U \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}$ is

a) 5 b) 5/2 c) 4 d) 3/2

MATRIX - MATCH TYPE

Each question contains statements given in two columns which have to be matched. Statements (A,B,C,D) in column I have to be matched with statements (p,q,r,s) in column II.

1. If A is non-singular matrix of order $n \times n$, then

Column - I

- A) $(\text{adj} A)^{-1} =$
 B) $\text{adj}(A^{-1}) =$
 C) $\text{adj}(kA) =$
 D) $\text{adj}(\text{adj} A) =$

Column - II

- p) $k^{n-1}(\text{adj} A)$
 q) $A/|A|$
 r) $|A|^{n-2}A$
 s) $\frac{\text{adj}(\text{adj}A)}{|A|^{n-1}}$

SECTION - II

(Multi Choice Single Correct)

1. If A' is the transpose of a square matrix A , then

- a) $|A| \neq |A'|$ b) $|A| = |A'|$ c) $|A| + |A'| = 0$
 d) $|A| = |A'|$ only when A is symmetric

2. If $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$, then B equals

- a) $I \cos\theta + J \sin\theta$ b) $I \sin\theta + J \cos\theta$ c) $I \cos\theta - J \sin\theta$
 d) $-I \cos\theta + J \sin\theta$

3. If I_n is the identity matrix of order n , then $(I_n)^{-1}$

- a) does not exist b) not equal to I_n c) equal to I_n d) none of these

4. If for a matrix $A, A^2 + I = 0$ where I is the identity matrix, then A equals

- a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ b) $\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$ c) $\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$ d) $\begin{pmatrix} -1 & 0 \\ - & -1 \end{pmatrix}$

5. If A and B are square matrices of the same order and A is non-singular then for a positive integer n , $(A^{-1}BA)^n$ is equal to

- a) $A^{-1}B^nA$ b) $n(A^{-1}BA)$ c) $A^{-n}B^nA^n$ d) none of these

6. If A and B are symmetric matrices of order n ($A \neq B$), then

- a) $A+B$ is skew symmetric b) $A+B$ is symmetric c) $A+B$ is a diagonal matrix
 d) $A+B$ is a zero matrix

7. If $A = \begin{pmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{pmatrix}$ and

- $B = \begin{pmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{pmatrix}$ then $AB = ?$
 a) A^3 b) B^2 c) O d) I

8. If A is a non-diagonal involutory matrix, then

- a) $A - I$ is non-zero singular b) $A + I = 0$ c) $A - I = 0$ d) none of these

9. The values of x for which the matrix $\begin{pmatrix} x+a & b & c \\ a & x+b & c \\ a & b & x+c \end{pmatrix}$ is non-singular are

- a) $R - \{0\}$ b) $R - \{-(a+b+c)\}$ c) $R - \{0, -(a+b+c)\}$ d) none of these

10. If $A = \begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$, then $19A^{-1}$ is equal to

- a) A' b) $2A$ c) $1/2A$ d) A