



# THE GURUKUL INSTITUTE

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D.P.P. of MATHEMATICS – PROBABILITY

## Intersection and Union of Sets of Events

- In a single cast with two fair dice, what is the chance of throwing  
(i) two 4's, (ii) a doublet, (iii) five – six, (iv) a sum of 7.
- Seven accidents occur in a week. What is the probability that they happen on the same day.
- From a bag containing 5 white, 7 red and 4 black balls a man draws 3 balls at random. Find the probability that all are white.
- From a pack of 52 cards two cards are drawn at random. Find the probability of the following events:  
(i) Both the cards are spades.  
(ii) One card is of spade and the other is a diamond.
- A has 3 shares in a lottery containing 3 prizes and 6 blanks ; B has one share in a lottery containing one prize and 2 blanks. Compare their chances of success.
- From a pack of cards, four are drawn at random. What is the chance that there is one card of each suit?
- In a given race the odds in favour of four horses A,B,C,D are 1:3, 1:4, 1:5, 1:6 respectively. Assuming that, a dead heat is impossible, find the chance that one of them wins the race.
- Ten passengers get into an elevator on the ground floor of a 20 floor building. What is the probability that they will all get out at different floors.
- There are three events A,B and C out of which one and only one can happen. The odds are 8 to 3 against A, 5 to 2 against B. Find odds against C.
- Out of  $(2n+1)$  tickets consecutively numbered from 1 to  $2n+1$ , three are drawn at random. Find the probability that the numbers on them are in A.P.

## OBJECTIVE

- A number of six digits is written down at random. Probability that sum of digits of the number is even is  
a)  $1/2$       b)  $3/8$       c)  $3/7$       d) none of these
- Two small squares on a chess board are chosen at random. Probability that they have a common side is,  
a)  $1/3$       b)  $1/9$       c)  $1/18$       d) none of these
- A bag contains 17 tickets numbered 1 to 17. A ticket is drawn and replaced, then one more ticket is drawn and replaced. Probability that first number drawn is even and second is odd is  
a)  $81/289$       b)  $72/289$       c)  $64/289$       d) none of these

## EXERCISE – 1

- If from a pack of 52 playing cards, one card is drawn at random, what is the probability that it is either a king or a queen.
- A bag contains 5 red, 4 green and 3 black balls. Three balls are drawn out from it at random. Find the probability of drawing exactly 2 red balls.
- 6 boys and 6 girls sit in a row at random. Find the probability that the boys and girls sit alternately.
- Find the odds in favour and odds against drawing 2 kings from a pack of 52 playing cards.
- What is the probability that four S's come consecutively if all the letters of the word 'MISSISSIPPI' are rearranged randomly?
- Six dice are thrown simultaneously, the chance of getting all possible faces is

- a)  $1/6$       b)  $1/36$       c)  $6!/6^6$       d)  $1/6!$

7. A bag contains 5 white, 7 black and 4 red balls. The probability that three balls drawn at random are all white is

- a)  $1/35$       b)  $1/56$       c)  $5/56$       d)  $5/28$

8. A bag contains 10 tickets numbered 1 to 10. Tickets are drawn one by one without replacement. The probability that the tickets number 7 is drawn at 4<sup>th</sup> draw is

- a)  $1/10$       b)  $1/10^4$       c)  $1/40$       d)  $2/5$

### CONDITIONAL PROBABILITY

1. If a dice is thrown, what is the probability of occurrence of a number greater than 1, if it is known that only odd numbers can come up.

2. In a college, 25% students failed in Mathematics, 15% students failed in Physics, and 10% students failed in Mathematics and Physics. A student is selected at random:

- (i) If he failed in Physics, then find the chance of his failure in mathematics,  
(ii) If he failed in Mathematics, then find the chance of his failure in Physics.  
(iii) Find the chance of his failure in Mathematics or Physics.

3. A set containing 'n' elements. A subset P of A is chosen at random. The set A is reconstructed by replacing the elements of P. A subset Q of A is chosen again at random. Find the probability that

- (i) P & Q have the same number of elements.  
(ii) The number of elements in P is more than the number of elements in Q.  
(iii) The number of elements in P is just one more than the number of elements in Q.  
(iv) Q is a subset of P.  
(v) P ∪ Q contains exactly r elements ( $1 \leq r \leq n$ ).

### OBJECTIVE

1. A and B play a game of tennis. The situation of the game is as follows ; if one scores two consecutive points after a deuce he wins; if loss of a point is followed by win of a point, it is deuce. The chance of a server to win a point is  $2/3$ . The game is at deuce and A is serving. Probability that A will win the game is, ( serves are changed after each point)

- a)  $3/5$       b)  $2/5$       c)  $1/2$       d)  $4/5$

2. A die is thrown three times and the sum of three numbers obtained is 15. The probability of first throw being 4 is

- a)  $1/18$       b)  $1/5$       c)  $4/5$       d)  $17/18$

3. Fifteen coupons are numbered 1,2,3.....15. Seven coupons are selected at random one at a time with replacement. The probability that the largest number appearing on the selected coupons is 9, is

- a)  $(9/16)^6$       b)  $(8/15)^7$       c)  $(3/5)^7$       d)  $(9^7 - 8^7) / 15^7$

4. A fair die is thrown until a score of less than 5 points is obtained. The probability of obtaining not less than 2 points on the last throw is

- a)  $3/4$       b)  $5/6$       c)  $4/5$       d)  $1/3$

### INDEPENDENT EVENTS

1. An event  $A_1$  can happen with probability  $p_1$  and event  $A_2$  can happen with probability  $p_2$ . What is the probability that

- (i) exactly one of them happens.  
(ii) at least one of them happens ( given  $A_1$  and  $A_2$  are independent events).

2. A person draws a card from a pack of 52, replaces it and shuffles it. He continues doing it until he draws a spade. What is the chance that he has to make

- (i) atleast 3 trials,      (ii) exactly 3 trials

3. A, B and C, in order, toss a coin. The one who gets a head first wins. Find their respective probabilities of winning.

## EXERCISE – 2

- Two dice are thrown. Find the probability that the sum of the numbers coming up on them is 9, if it is known that the number 5 always occurs on the first dice.
- A card is drawn at random from a pack of cards. What is the probability that it is a king if it is known that a red card is drawn.
- If the odds against winning a race of three horses are respectively 3:1, 4:1, and 5:1, what is the probability that one of these horses will win? ( Assume a dead heat is impossible)
- A and B are two independent events. The probability that both A and B occur is  $\frac{1}{6}$  and the probability that neither of them occurs is  $\frac{1}{3}$ . Find the probability of occurrence of A.
- A card is drawn from an ordinary pack and a gambler bets that it is a spade or an ace. The odds against his winning the bet is
  - 4:9
  - 9:4
  - 2:3
  - none of these
- The letters of word ASSASSIN are written at random. The probability that no two S are together is
  - $\frac{1}{14}$
  - $\frac{5}{14}$
  - $\frac{1}{4}$
  - $\frac{1}{2}$
- A number is chosen at random from the numbers 10 to 99. By seeing the number a man will laugh if product of the digits is 12. If he chooses three numbers with replacement then the probability that he will laugh at least once is
  - $1 - \left(\frac{3}{5}\right)^3$
  - $\left(\frac{43}{45}\right)^3$
  - $1 - \left(\frac{4}{25}\right)^3$
  - $1 - \left(\frac{43}{45}\right)^3$

## The Probability Theorem

- A bag contains 3 white balls and 2 black balls, another contains 5 white and 3 black balls. If a bag is chosen at random and a ball is drawn from it, what is the probability that it is white?
- Find the probability that a year chosen at random has 53 Sundays.

## Baye's Theorem

- Each of three bags A, B, C contains white balls and black balls. A has  $a_1$  white &  $b_1$  black, B has  $a_2$  white &  $b_2$  black and C has  $a_3$  white &  $b_3$  black. A ball is drawn at random and is found to be white. Find the respective probability that it is from A, B & C.
- A bag contains 5 balls and of these it is equally likely that 0, 1, 2, 3, 4, 5 are white. A ball is drawn and is found to be white. What is the chance that it is the only white ball?

## PROBLEMS

- A bag contains 10 white balls as well as black. A person draws 6 balls and find that 3 balls are white and the other three are black. Find the probability that the number of white balls in the bag is the same as that of black balls.
- A letter is known to have come either from TATANAGAR OR CALCUTTA. On the envelope just two consecutive letters TA are visible. What is the probability that the letter came from CALCUTTA?
- A pack of card has one card missing. Two cards are drawn and are found to be spades. What is the probability that the missing card is not a spade?

## OBJECTIVE

- If the papers of 4 students can be checked by any one of the 7 teachers, then the probability that all the 4 papers are checked by exactly 2 teachers is:
  - $\frac{2}{7}$
  - $\frac{32}{343}$
  - $\frac{12}{49}$
  - $\frac{6}{49}$

## EXERCISE – 3

- A man throws a die three times. If in first throw an odd prime occurs then he adds the numbers appearing in next two draws otherwise he multiplies them. Find the probability that number thus obtained is an even number.
- A man is known to speak the truth 5 out of 7 times. He throws a die and reports that it is an even number. Find the probability that it is actually an even number.

3. There are three bags containing respectively 1 white, 2 red, 3 black balls; 2 white, 3 red, 1 black balls and 3 white, 1 red and 2 black balls. A bag is chosen at random and from it two balls are drawn at random. The two balls are one red and one white. What is the chance that they came from the (a) first bag (b) second bag (c) third bag
4. A bag A has 3 red and 2 black balls, and a bag B has 3 red and 4 black balls. Then one ball is drawn from B and placed in A. If one ball is drawn from A, what is the probability that it is red?
5. Suppose there are 3 groups A, B and C comprising 4 girls, 3 boys; 6 girls, 7 boys and 3 girls, 2 boys, respectively. One child is selected at random from each group. The probability that the 3 selected are 2 girls and 1 boy is  
 a)  $186/455$     b)  $86/355$     c)  $186/255$     d) none of these
6. An anti-aircraft gun can take maximum of four shots at an enemy plane moving away from it. The probabilities of hitting the plane at the first, second, third and fourth shot are 0.4, 0.3, 0.2, and 0.2 respectively. The probability that the plane gets hit is  
 a) 0.3875    b) 0.7312    c) 0.5    d) none of these
7. Two cards are missing from a pack of cards, one card is drawn. The probability that this card is a king is  
 a)  $1/13$     b)  $2/13$     c)  $25/26$     d) none of these

### **BINOMIAL DISTRIBUTION FOR SUCCESSIVE EVENTS**

1. A die is thrown 7 times. What is the chance that an odd no. turns up  
 (i) exactly 4 times    (ii) at least 4 times.
2. A and B play a series of games which cannot be drawn and p, q are their respective chances of winning a single game. What is the chance that A wins m games before B wins n games.
3. If a coin is tossed n times, what is the probability that head will appear an odd number of times.
4. A bag contains n white and n black balls, all of equal size. Balls are drawn at random. Find the probability that there are both white and black balls in the draw and that the number of white balls is greater than those of black balls by one.

### **EXERCISE – 4**

1. If a dice is thrown 5 times. What is the chances that an even number will come up exactly 3 times.
2. If two dice are thrown 5 times, what is the probability that the sum of the numbers coming up will be 10 in exactly 2 out of these 5 trials.
3. Five shots are fired at a target. If each shots has a probability 0.6 of hitting the target, what is the probability that the target will be hit at least once.
4. The probability that England wins a cricket match against India is  $1/3$ . If India and England play 3 test matches, what is the probability that India will win 2 test matches.
5. A special die with numbers 1, -1, 2, -2, 0 and 3 is thrown thrice. The probability that the total is 6 is  
 a)  $1/108$     b)  $25/216$     c)  $5/216$     d)  $5/108$
6. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to getting 9 heads, then the probability of getting 2 heads is,  
 a)  $15/2^8$     b)  $2/15$     c)  $15/2^{13}$     d) none of these
7. Three cards are drawn from a pack of cards. The chance of getting different suit as well as different denominations is  
 a)  $132/425$     b)  $123/425$     c)  $132/245$     d) none of these
8. In a tournament with 5 teams each team plays every other team once. Each game ends in a win for one of the two teams. Each team has  $1/2$  chance of winning each game. The probability that no team wins all its games or loses all its games is  
 a)  $5/32$     b)  $27/32$     c)  $15/32$     d)  $17/32$

## ASSIGNMENT

### SECTION – I ( PART – A)

1. Let  $A = \{1,2,3,4,5,6\}$ . Two numbers are drawn with replacement from the set A. What is the probability that one of them is odd and other is even.
2. From a bag containing 8 white and 5 black balls, four balls are drawn one by one without replacement, find the probability that exactly two of them are white.
3. If the letters of the word REGULATIONS be arranged at random, then find the probability that there are exactly four letters between R and E.
4. If four coins are tossed, find the probability that there shall be two heads and two tails.
5. A fair die is rolled four times. Find the probability that each number is no smaller than the preceding number.
6. A can solve 75% of the problems in a book on mathematics and B can solve 70%. What is the probability that a problem chosen at random is solved?
7. Five coins whose faces are marked 2,3 are thrown. What is the chance of obtaining a total of 12?
8. Five persons a,b,c,d,e are contesting in an election. Three persons are to be elected. If one of them d, has been elected uncontested, then find the probability that c would be elected.
9. If A and B are two independent events, prove that  $P(A \cup B) \cdot P(A' \cap B') \leq P(C)$ , where C is an event defined that exactly one of A and B occurs.
10. (i) The chance of success of an event is  $\frac{3}{4}$ ; find the probability that in six trials there will be four or more successes.  
(ii) I take a step forward with probability 0.45 and back with a probability 0.55. What is the probability that after 13 steps I am 3 steps away from the starting point?
11. Probability of A winning a game is 0.4 while that of B is 0.6. What is the probability of A winning, if (i) they play the "best of 7" games, (ii) they play the "best of 11" games.  
(All the games are played)
12. F is the set of all onto functions from  $A = \{a_1, a_2, \dots, a_n\}$  to  $B = \{x, y, z\}$  and  $f \in F$  is chosen randomly. What is the probability that x is the image of two elements in A.
13. There are three similar coins, one of which is ideal and other two are biased. The chances of head are respectively  $\frac{1}{2}$ ,  $\frac{1}{3}$  and  $\frac{2}{3}$ . A coin is selected at random and tossed twice. If head occurs both times, find the probability that the ideal coin was selected.

### PART – B (Multi Choice Single Correct)

1. If  $P(A/B) = P(B/A)$ . A and B are two non – mutually exclusive events then  
(a) A and B are necessarily same events (b)  $P(A) = P(B)$   
(c)  $P(A \cap B) = P(A) P(B)$  (d) all the above
2. If Ram and Shyam selected two numbers with replacement from the set 1 to n. If the probability that Shyam selects a number which is less than the number selected by Ram is  $\frac{63}{128}$ , then  
a) n is odd b) n is a perfect square c) n is perfect 4<sup>th</sup> power d) none of these
3. Three distinct numbers are chosen from the set  $\{1,2,\dots,6\}$ . The probability that the product of two numbers is the third one is  
a)  $\frac{1}{60}$  b)  $\frac{1}{30}$  c)  $\frac{1}{20}$  d) none of these
4. A and B are two events such that  $P(A)=0.2$  and  $P(A \cup B) = 0.7$ . If A and B are independent events then  $P(B)$  equals  
a)  $\frac{2}{7}$  b)  $\frac{7}{9}$  c)  $\frac{5}{8}$  d) none of these

5. A die is thrown a fixed number of times. If probability of getting even number 3 times is same as the probability of getting even number 4 times, then probability of getting even number exactly once is  
 a)  $1/4$       b)  $3/128$       c)  $5/64$       d)  $7/128$
6. If 'head' means one and 'tail' means two, then coefficients of quadratic equation  $ax^2+bx+c=0$  are chosen by tossing three fair coins. The probability that roots of the equations are imaginary is  
 a)  $5/8$       b)  $3/8$       c)  $7/8$       d)  $1/8$
7. If two events A and B are such that  $P(A) > 0$  and  $P(B) \neq 1$ , then  $P(A/B)$  is equal to  
 a)  $1 - P(A/B)$       b)  $1 - \overline{P(A/B)}$       c)  $1 - \overline{P(A \cup B)} / P(B)$       d)  $P(A) / P(B)$
8. The probability of having at least one tail in 4 throws with a coin is  
 a)  $15/16$       b)  $1/16$       c)  $1/4$       d)  $1$
9. One boy can solve 60% of the problems in a book and another can solve 80%. The probability that at least one of the two can solve a problem chosen at random from the book is  
 a)  $2/25$       b)  $23/25$       c)  $4/5$       d)  $9/10$
10. Three dice are rolled. The probability that different numbers will appear on them is  
 a)  $2/3$       b)  $4/9$       c)  $5/9$       d)  $2/9$
11. In a bag there are 15 red and 5 white balls. Two balls are chosen at random and one is found to be red. The probability that the second one is also red is  
 a)  $12/19$       b)  $13/19$       c)  $7/12$       d)  $15/19$
12. Three dice are thrown simultaneously. The probability that 4 has appeared on two dice given that 5 has occurred on one dice is  
 a)  $1/6$       b)  $3/91$       c)  $3/216$       d)  $1/36$
13. A jar contains initially one amoeba. Every minute, every amoeba either dies or does nothing or splits into 2 or splits into 3 amoebae, with probability  $1/4$  of each event. The probability that amoeba population eventually dies out within two minutes, is  
 a)  $6/16$       b)  $1/4$       c)  $10/16$       d) none of these
14. A die is thrown n times ( n being odd). The probability that even face turns odd number of times, is  
 a)  $n/(2n+1)$       b) less than  $1/2$       c)  $1/2$       d) greater than  $1/2$

### Multi Choice Multi Correct

1. An integer is chosen at random from first two hundred natural numbers then the probability that integer is either divisible by 2, 6 or 8 is  
 a)  $1/2$       b)  $1/4$       c)  $29/100$       d)  $> 1/4$
2. A fair coin is tossed 9 times the probability that at least 5 consecutive heads occurs is  
 a)  $5/64$       b)  $3/32$       c)  ${}^9C_5(1/2)^9$       d)  ${}^3C_1 / {}^{32}C_1$
3. Let E and F be two independent events. The probability that both E and F happen is  $1/12$  and the probability that neither E nor F happen is  $1/2$ , then  
 a)  $P(E) = 1/3, P(F) = 1/4$       b)  $P(E) = 1/2, P(F) = 1/6$       c)  $P(E) = 1/6, P(F) = 1/2$       d)  $P(E) = 1/4, P(F) = 1/3$

### Numerical Based Type

1. The probability that a man can hit a target is  $3/4$  and the probability that he will hit at least n – 1 times is  $189/256$ , then the number of times, n he tried is
2. The minimum number of tosses of a pair of dice so that the probability of getting the sum of the digits on the dice equal to 7 on at least one toss is greater than 0.95, is n, then  $n+1/6$  is?
3. Consider a bag containing 10 balls of which a few are black balls. Probability that bag contains exactly 3 black balls is 0.6 and probability of bag containing exactly 1 black ball is 0.4. Now, balls are drawn from the bag, one at a time, without replacement, till all black balls have been drawn. The probability that this procedure would end at the 6<sup>th</sup> draw is p, then 100 p is equal to?

### LINKED COMPREHENSION

Read the following write up carefully and answer the following questions:

There are two die A and B both having six faces. Die A has three faces marked with 1, two faces marked with 2 and one face marked with 3. Die B has one face marked with 1, two faces marked with 2 and three faces marked with 3. Both dice are thrown randomly once. If E be the event of getting sum of the numbers appearing on top faces equal to x and let P(E) be the probability of event E.

- P(E) is maximum when x equal to  
a) 5                      b)3                      c)4                      d) 6
- P(E) is minimum when x equals to  
a) 3                      b)4                      c)5                      d)6
- When  $x=4$ , then P(E) is equal to  
a)  $5/9$                       b)  $6/7$                       c)  $7/18$                       d)  $8/19$

### SECTION – II ( Multi Choice Single Correct)

- A and B are two independent events. The probability that A and B happen simultaneously is  $1/12$  and neither A nor B happens is  $1/2$ , then  
a)  $P(A)=1/3, P(B)=1/4$     b)  $P(A)=1/2, P(B)=1/6$     c)  $P(A)=1/5, P(B)=1/3$     d)  $P(A)=1/6, P(B)=1/2$
- Entries of a  $2 \times 2$  determinant are chosen from the set  $\{-1, 1\}$ . The probability that determinant has zero value is  
a)  $1/4$                       b)  $1/3$                       c)  $1/2$                       d) none of these
- Two events A and B have probabilities 0.25 and 0.50 respectively. The probabilities that both A and B occur simultaneously is 0.14. Then the probability that neither A nor B occurs is  
a) 0.39                      b) 0.25                      c) 0.11                      d) none of these
- The probability that at least one of the events A and B occurs is 0.6. If A and B occur simultaneously with probability 0.2, then  $P(A) + P(B)$  is  
a) 0.4                      b) 0.8                      c) 1.2                      d) 1.4
- Let A, B, C be 3 independent events such that  $P(A)=1/3, P(B)=1/2, P(C)=1/4$ . Then probability of exactly 2 events occurring out of 3 events is  
a)  $1/4$                       b)  $9/24$                       c)  $3/4$                       d) none of these
- Three different dice are rolled three times. The probability that they show different numbers only two times is  
a)  $1/3$                       b)  $({}^6P_3/6^3)^2$                       c)  $107/(54)^3$                       d)  $100/243$
- If sum of the ordinate and the abscissa of a point P(x,y) is  $2n$  (x,y are natural numbers), then the probability that P does not lie on the line  $y=x$  is  
a)  $(n-1)/(n+3)$                       b)  ${}^{2n}C_n / 2^{2n}$                       c)  $(2n-2)/2n-1$                       d)  $(2n+1)/(2n+3)$
- If two subsets A and B of set S containing n elements are selected at random, then the probability that  $A \cap B = \phi$  and  $A \cup B = S$  is  
a)  $1/2$                       b)  $1/2^n$                       c)  $(3/4)^4$                       d)  $1/3^n$
- A speaks truth in 60% cases and B speaks truth in 70% cases. The probability that they will say the same thing while describing single event is  
a) 0.56                      b) 0.54                      c) 0.38                      d) 0.94