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Progression & Series

Arithmetic Progression

1.If 1^{st} and 2^{nd} terms of an A.P. are 1 and -3 respectively, find the n^{th} term and the sun of the first n terms.

2.If 6 arithmetic means are inserted between 1 and 9/2, find the 4^{th} arithmetic mean.

3.(i) Find the A.P. whose 7^{th} and 13^{th} terms are respectively 34 and 64.

(ii) If a,b,c are in A.P., prove that b+c, c+a, a+b are also in A.P.

(iii)The sum of four integers in A.P. is 24 and their product is 95. Find the numbers.

(iv) The first, second and last terms of an A.P. are a,b,c respectively. Then prove that the sum is

$$\frac{(a+c)(b+c-2a)}{2(b-a)}$$

GEOMETRIC PROGRESSION

4.The 7^{th} term of a G.P. is 8 times the 4^{th} term. Find the G.P. when its 5^{th} term is 48.

5.Does there exist a G.P. containing 27,8, and 12 as three of its terms? If it exists, how many such progressions are possible?

6.(i)If the $(p+q)^{th}$ term of a G.P. is a and the $(p-q)^{th}$ term is b, show that its p^{th} term is $\sqrt{ab}$.

(ii)How many terms of the series $1+3+3^2+3^3+$…………..$+3^{n-1}$ must be taken to make the sum equal to 3280.

(iii) Find the sum of n terms of the series $(a+b)+(a^2+2b)+(a^3+3b)+$…………….

(iv) If the product of three consecutive numbers in G.P. is 216 and the sum of the product taken in pairs is 156, then find the numbers.

ARITHMETIC – GEOMETRIC PROGRESSIO

7.Find the sum of the series $1 . 2+2 . 2^2+3 . 2^3+$…………..$+ 100 . 2^{100}$.

8.Find the sum of the series

(i) $1+ 2x +3x^2+4x^3+$……

(ii) $n+(n-1)x + (n-2)x^2+$…….$+ 2x^{n-2}+ x^{n-1}$.

HARMONIC Progression

9. Find the 4^{th} and 8^{th} term of the series 6,4,3,……..

10. If a, b, c are in H.P. show that $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are also in H.P.

11.(i)Find the 4^{th} term of an H.P., whose 7^{th} term is 1/20 and 13^{th} term is 1/38.

(ii) If a,b,c are in H.P., show that $\frac{a-b}{b-c} = \frac{a}{c}$

(iii) If a^2, b^2, c^2 are in A.P./ then show that b+c,c+a,a+b are in H.P.

(iv)If a,b,c are in H.P., prove that $\left(\frac{1}{a}+\frac{1}{b}-\frac{1}{c}\right)\left(\frac{1}{b}+\frac{1}{c}-\frac{1}{a}\right)=\left(\frac{4}{ac}-\frac{3}{b^2}\right)$

SOME IMPORTANT RESULTS

12. Find the n^{th} term and the sum of n terms of the series 1 . 2 . 4+ 2 . 3 . 5+ 3 . 4 . 6+……………..

13.Find the sum of the series $1 . n+2(n-1)+3.(n-2)+$………$+ n . 1$

METHOD OF DIFFERENCES

14.Find the sum of I^{st} n terms of the series 5,7,11,17,25…………

15.Find the sum of the series $\frac{1}{1.2.3.4} + \frac{1}{2.3.4.5} + \ldots$ n tem

16.Find the sum of the series 1.2.3.4.5 + 2.3.4.5.6+…………n terms.

17.Find the sum to n terms of series

(i) 2+6+12+20+……………

(ii) 3+5+9+17+33+…………

(iii) $\frac{1}{1.3.5} + \frac{1}{3.5.7} + \ldots$

(iv) 1.3.5.7 + 3.5.7.9+………..

INEQUALITIES

18. If x,y,z are positive real numbers,such that x+y+z=a. then prove that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} \geq \frac{9}{a}$.

19.Prove that (a+b+c)(ab+bc+ca) > 9abc.

20.If a,b,c are positive real numbers such that a+b+c=18, find the maximum value of $a^2b^3c^4$.

21.If a,b,c are positive real numbers such that a + b+ c=1, then prove that :

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}.$$

22.(i)Show that $x + \frac{1}{x} \geq 2$, if x>0 and $x + \frac{1}{x} \leq -2$, if x<0.

(ii)Show that $\frac{a_1}{a_2} + \frac{a_2}{a_3} + \frac{a_3}{a_4} + \ldots \frac{a_{n-1}}{a_n} + \frac{a_n}{a_1} > n$, where $a_1, a_2, \ldots, a_n$ are different positive integers

(iii)Show that $b^2c^2 + c^2a^2 + a^2b^2 > abc\,(a+b+c)$, where a,b,c are different positive integers.

(iv)Find the greatest value of x^3y^4 if 2x + 3y =7 and x≥0,y≥0.

(v)If a,b,c are unequal and positive, show that $\frac{bc}{b+c} + \frac{ca}{c+a} + \frac{ab}{a+b} < \frac{1}{2}(a+b+c)$.

PROBLEMS

1.The interior angles of a polygon are in arithmetic progression. The smallest angle is 120^0 and the common difference is 5^0.Find the number of sides of the polygon.

2.If $a_1, a_2, \ldots, a_n$ are in A.P. (a_i>0 for all i).

Show that $\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \ldots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{n-1}{\sqrt{a_1} + \sqrt{a_n}}$

3.Find the numbers a,b,c between 2 and 18 such that

(i) their sum is 25,

(ii) the numbers 2,a,b are consecutive terms of an A.P. and

(iii) the numbers b,c,18 are consecutive terms of a G.P.

4.If the $(m+1)^{th}$, $(n+1)^{th}$ and $(r+1)^{th}$ term of an A.P. are in G.P., m, n,r, are in H.P., show that the ratios of the common difference to the first term in the A.P. is (-2/n).

5.If the m^{th},n^{th}, and p^{th} terms of an A.P. and a G.P. be equal and be respectively x,y,z, then prove that $x^{y-z}. y^{z-x}.z^{x-y} = 1$ or $x^y\, y^z\, z^x = x^z . y^x . z^y$.

6.Let $x=1+3a+6a^2+10a^3+\ldots$ to ∞ (|a|<1).

$y = 1+4b+10b^2+20b^3+\ldots$ to ∞ (|b|<1).

Find $S=1+3(ab)+ 5(ab)^2 + \ldots$ to ∞ in terms of x and y.

7.If the equation $x^4 - 4x^3+ax^2+bx+1=0$ has four positive roots, then find a and b.

8.Find the sum of the series $\frac{1}{1.3} + \frac{2}{1.3.5} + \frac{3}{1.3.5.7} + \ldots\ldots$n terms.

9.Give the perimeter of a triangle, prove that the triangle of the greatest area is equilateral.

10.If a,b,c are the positive real numbers, prove that $a^2(1+b^2)+b^2(1+c^2)+c^2(1+a^2) \geq 6abc$.

OBJECTIVE

1. The third term of a G.P. is 4, the product of the first five terms is

   a) 4^3 b) 4^5 c) 4^4 d) none of these

2.If a,b,c,d and p are distinct real numbers such that
$(a^2+b^2+c^2)p^2 - 2(ab+bc+cd)p+(b^2+c^2d^2) \leq 0$, then a,b,c,d are in

   a) A.P. b) G.P. c)H.P. d) none of these

3.Coefficient of x^{99} in the polynomial $(x-1)(x-2)\ldots..(x-100)$ is

   a) 100! b) -5050 c) 5050 d) – 100!

4.The sum of the first n terms of the series
$1^2+2 \cdot 2^2+3^2+2 \cdot 4^2+5^2+2 \cdot 6^2+\ldots\ldots$ is $n(n+1)^2$, when n is even.
When n is odd, the sum is

a) $\frac{n^2(n+1)}{2}$ b) $\frac{n(n+1)(2n+1)}{6}$ c) $\frac{n(n+1)^2}{2}$ d) $\frac{n^2(n+1)^2}{2}$

5.Given p A.P's, each of which consists of n terms. If their first terms are 1,2,3………,p and common difference are 1,3,5,……….,2p – 1 respectively, than sum of the terms of all the progressions is

   (A) 1/2np(np+1) (B) 1/2n(p+1) (C) np(n-1)
   (D) none of these

6. If p, q, r are in A.P., then p^{th}, q^{th} and r^{th} terms of any G.P. are in

   (A) A.P. (B) G.P. (C) H.P. (D) A.G.P.

7.If $4a^2+9b^2+16c^2 = 2(3ab+6bc+4ac)$, where a,b,c, are non- zero numbers, then a,b,c, are in

   a)A.P. b) G.P. c)H.P. d) none of these

8.If a,b and c are positive real numbers, then $\frac{a}{b} + \frac{b}{c} + \frac{c}{a}$ is greater than or equal to

   a)3 b) 6 c)27 d) none of these

9.If a,b and c are distinct positive real numbers and $a^2+b^2+c^2=1$, then ab+bc+ca is

   a) less than 1 b) equal to 1 c) greater than 1
   d) any real number

10.If the product of n positive numbers is unity, then their sum is

   a) a positive integer b) divisible by n
   c) equal to n+1/n d) never less than n.

11.If a,b and c are positive real numbers, then the least value of $(a+b+c)\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$ is

   a) 9 b) 3 c) 10/3 d) none of these

12.Ift he first and the (2n – 1)th terms of an A.P., G.P. and H.P are equal and their n^{th} terms are a,b,c respectively, then

   a) a+c=2b b)a+c=b c) $ac - b^2=0$ d) none of these

13. If a,b and c are positive real numbers, then a + b + c is greater than or equal to

   a) 3 b) 6 c) 26 d) none of these

14. If a,b,c and d are different positive numbers in H.P., then

   a) a+b>c+d b)a+c>b+d c) a+d>b+c d) none of these

ASSIGNMENTS
SECTION – I
PART – A (LEVEL – I)

1.Find the sum of the integers between 1 and 200 which are multiples of 3 and 7.

2.The 3^{rd} term of an A.P. is 7 and its 7^{th} term is 2 more than thrice of its 3^{rd} term. Find the first term, common difference and sum of its first 20 terms.

3.In a set of four numbers, the first three are in G.P. and the last three in A.P. with common difference 6. If the first number is the same as the fourth, find the four numbers.

4.If the ratio of the sum of m terms and n terms of an A.P. be $m^2:n^2$, prove that the ratio of its m^{th} and n^{th} terms will be $(2m – 1): (2n – 1)$.

5.The harmonic mean of two numbers is 4, and their arithmetic mean A, and geometric mean G satisfy the relation $2A+G^2=27$. Find the two numbers.

6.In an increasing G.P., the sum of the first and the last is 66, the product of the second and the last but one term is 128, and the sum of all the terms is 126, how many terms are there in the progression?

7.If $x=1+a+a^2+a^3+$........ to ∞ ($|a|<1$), $y=1+b+b^2+b^3+$............. to ∞ ($|b|<1$).

Prove that $1+ ab+ a^2b^2 + a^3b^3+$. to $\infty = \dfrac{xy}{X+y-1}$

8.The p^{th} term of an A.P. is a and q^{th} term is b.

Prove that sum of its (p+q) terms is $\dfrac{p+q}{2}\left(\dfrac{a+b+a-b}{p-q}\right)$

9.The sum of three consecutive terms in H.P. is 37 and the sum of their reciprocal is 1/4. Find the numbers.

10. Find the sum of n terms of the series $1^3 + 3^3 + 5^3 +$........

LEVEL – II

1.If a,b,c are in H.P., then prove that $\dfrac{a}{b+c-a}, \dfrac{b}{c+a-b}, \dfrac{c}{a+b-c}$ are in H.P.

2.(i)Sum upto n terms the series 6+66+666+....

(ii)Express the recurrent decimal 0.125125125.......as a rational number.

(iii)Sum the series 1.1!+ 2.2!+3.3!+4.4! +........ to n terms.

3.If a,b,c are in A.P and a,b,d are in G.P, prove that a, a – b, d – c are in G.P.

4.If $\log_3 2$, $\log_3 (2^x – 5)$ and $\log_3 (2^x – 7/2)$ are in A.P., then find the value of x.

5.Let a_n be the n^{th} term of an A.P. show that

$$a_1^{\,2} - a_2^{\,2} + a_3^{\,2} - a_4^{\,2} + \ldots\ldots + a^2_{2n-1} - a^2_{2n} = \frac{n}{2n-1}\left(a_1^{\,2} - a^2_{2n}\right)$$

6. Find the sum of the series

(i) 1+3+7+15+31+....... to n terms.

(ii) 1+5+11+19+29+.......to n terms.

(iii)$1 . 2+2 . 3x+ 3 . 4x^2+$..........∞ ($|x| <1$)

(iv) $3+5x+9x^2+15x^3+23x^4+33x^5+$...........$\infty$ ($|x|<1$)

7.If 9 Arithmetic and harmonic means be inserted between 2 and 3, prove that A+6/H=5, where A is any of the arithmetic means and H is the corresponding harmonic mean.

8.If $x_1,x_2,x_3.....x_n$ are in H.P, then prove that $x_1x_2 + x_2x_3 + x_3x_4 +.....+ x_{n-1}x_n=(n – 1)x_1x_n$.

9.(i)If $a_1,a_2,......a_n$ are n positive real numbers such that $a_1.a_2..........a_n=1$,show that $(1+a_1)(1+a_2)......(1+a_n)\geq 2^n$.

(ii)If n is a positive integer,prove that $2^n>1+ n\sqrt{2^{n-1}}$;n>1.

10.N, the set of natural numbers, is partitioned into subsets $S_1=\{1\}$, $S_2=\{2,3\}$, $S_3=\{4,5,6\}$, $S_4=\{7,8,9,10\}$ and so on. Find the sum of the elements in the Subset S_{50}.

11.Show that $(1+3^{-1})(1+3^{-2})(1+3^{-4})(1+3^{-8})\ldots\ldots\ldots(1+3^{-2n}) = 3(1-3^{-2n-1})$.

12.The fourth power of the common difference of an arithmetic progression with integer entries is added the product of any four consecutive terms of it. Prove that the resulting sum is the square of an integer.

13.(i)Prove that $a^4 + b^4 + c^4 > abc(a+b+c)$, where a,b,c are different positive real numbers.

(ii)Prove that ab + bc +ca > 1 + 1 + 1, where a,b,c are different positive real numbers.

14.Find the sum of n terms. Also find the sum to infinite terms

(i) $1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \ldots\ldots\ldots\ldots\ldots$ b) $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \ldots\ldots\ldots\ldots\ldots\ldots$

PART – B

Multiple Choice Questions (Single Option Correct)

1.If $S_n = 1+3+6+10+\ldots\ldots\ldots\frac{n(n+1)}{2}$ then S_n is

a) $\frac{1}{6}n(n+1)(n+2)$ b) $\frac{1}{6}n(n+1(2n+1)$ c) $\frac{1}{4}n^2(n+1)^2$ d)none of these

2.If x,2x+2,3x+3 are in G.P., then the fourth term is

a)27 b) -27 c)13.5 d) -13.5

3.Let the harmonic mean and geometric mean of two positive numbers be in the ratio 4:5. Then the two numbers are in the ratio

a) 1:4 b) 1:2 c) 4:5 d) none of these

4.The sum of the integers from 1 to 100 that are divisible by 2 or 5 is

a) 3000 b) 3050 c) 3600 d) none of these

5.If the sum of first n natural numbers is 1/5 times the sum of their squares, then the value of n is

a)7 b) 6 c)8 d) none of these

6.If two arithmetic means A_1,A_2, two geometric means G_1,G_2 and two harmonic means H_1,H_2 are inserted between any two numbers, then $\frac{A_1 + A_2}{H_1 + H_2}$ is

a) $\frac{G_1G_2}{H_1H_2}$ b)$\sqrt{G_1G_2}$ c) $\frac{H_1H_2}{G_1G_2}$ d) none of these

7.If S be the sum, p the product and R the sum of the reciprocals of n terms of a G.P., then $(S/R)^n$ is equal to

a) p^2 b) p^3 c)p d) none of these

8.For n≥3, the n roots of unity form

a) H.P. b) an A.P. c) a G.P. d) none of these

9.The rational number which equals to the number 2.357 with recurring decimal is

a) 2355 b)2370 c) 2355 d) none of these

10.If $\frac{3+5+7+\ldots\ldots\ldots+ \text{n terms}}{5+8+11+\ldots\ldots+10 \text{ terms}} = 7$, then the value of n is

a) 35 b) 36 c) 37 d)40

11.If x,y,z are in G.P. and $a^x = b^y = c^z$ then

a) $\log_b a = \log_a c$ b) $\log_c b = \log_a c$ c) $\log_b a = \log_c b$ d) none of these

12.The harmonic mean of the roots of the equation $(5+\sqrt{2})x^2 - (+\sqrt{5})x+8+2\sqrt{5} =0$ is

a) 2 b) 6 c) 4 d) none of these

13.If $\frac{a^n + b^n}{a^{n-1}+b^{n-1}}$ is the GM between a and b, then the value of n is

a) 0 b)1 c) 1/2 d) none of tehse

14.If $\frac{a^n + b^n}{a^{n-1}+b^{n-1}}$ is the AM between a and b, then the value of n is

a)0 b)1 c) -1 d) none of these

15.If three positive real numbers a,b,c are in A.P., with abc=4, then the minimum value of b is

a) $4^{1/3}$ b) 3 c) 2 d) 1/2

16.If a,b and c are three positive real numbers, then the minimum value of the expression $\frac{b+c}{a} + \frac{c+a}{b} + \frac{a+b}{c}$ is:

a) 1 b)2 c)3 d)6

17.The least value of the expression $5^{\sin x-1} + 5^{-\sin x-1}$ is

a) 2/5 b) 1/5 c) 5 d) 5/2

18.If $a_1, a_2, a_3, \ldots$ is an A.P. such that $a_1+a_5+a_{10}+a_{15}+a_{20}+a_{24} = 225$, then $a_1+a_2+a_3+\ldots+a_{23}+a_{24}$ is equal to

a) 909 b) 75 c) 750 d) 900

19.The sum of the series $1+2(1+1/n)+3(1+1/n)^2+\ldots\ldots\infty$ is given by

a) n^2+1 b) $n(n+1)$ c) $n(1+1/n)^2$ d) n^2

20.All the terms of an A.P. are natural numbers. The sum of its first nine terms lies between 200 and 220. If the second term is 12, then the common difference is

a)2 b) 3 c) 4 d) none of these

Multiple Choice Questions(Multiple Options correct)

1.In a G.P. the product of the first four terms is 4 and the second term is the reciprocal of the fourth term. The sum of infinite terms of the G.P. is

a) -8 b) -8/3 c) 8/3 d)8

2.Let a,x,b be in A.P., a,y b be in G.P. and a,z,b be in H.P. If x=y+2 and 2=5z, then

a) $y^2=xz$ b) x>y>z c) a=9, b=1 d) a=1/4, b=9/4

3If three unequal positive real numbers a,b,c are in G.P. and b – c, c – a, a – b are in H.P., then the value of a+b+c depends on

a) a b) b c) c d) none of these

Match The Following

List – I	List – II
a) In an infinite G.P. each term is equal to the sum of all the succeeding terms.The common ratio of the G.P. is	p) 4
	q) 1/2
b)If $(1 - \sqrt{2})^n$, $(1+ \sqrt{2})^n$ are in G.P., then n can be	r) 7
c)The least value of n for which $1+3+3^2+3^3+\ldots\ldots+3^{n-1} > 1000$	s) 1/4
d) If $3+5r+7r^2+\ldots\ldots\infty$ is 44/9, then r is equal to	t) 2

Numerical Based

1.Find the sum of infinite terms of the series $\frac{3}{1^2} + \frac{5}{1^2+2^2} + \frac{7}{1^2+2^2+3^2} + \frac{9}{1^2+2^2+3^2+4^2} + \ldots\ldots$

2.The least value of $2\log_{100} a - \log_a 0.0001$, a>1 is?

SECTION – II

1.If a,b,c are the p^{th},q^{th} and r^{th} terms respectively of an H.P., then ab(p – q)+bc(q – r)+ca(r - p) equals to

a) 1 b) -1 c) 0 d) none of these

2.If a,b,c,d are in GP and $a^x=b^y=c^z=d^u$, then x,y,z,u are in

a) AP　　b) GP　　c)HP　　d) none of these

3.The sum of first n terms of the series 1 + 3 + 7 +15 +……… is equal to

a) $2^n - n - 1$　　b) $1 - 2^n$　　c)$n+2^{-n} - 1$　　d) $2^n - 1$

4.If $1+a+a^2+a^3+\ldots\ldots a^n = (1+a)(1+a^2)(1+a^4)$, then n is equal to

a) 3　　b) 5　　c) 7　　d)9

5.If a,b,c and d are in H.P., then ab+bc+cd is equal to

a) 3ad　　b) (a+b)(c+d)　　c) 3ac　　d) none of these

6.The sum of all possible products of first n natural numbers taken two at a time, is

a) $1/24n(n+1)(n-1)(3n+2)$

b)$1/6n(n+1)(n-1)(2n+2)$

c)$1/24\, n(n-1)(n-1)(2n+3)$

d) none of these

7.If a>1,b>1, then the minimum value of $\log_b a+\log_a b$ is

a)0　　b)1　　c)2　　d) none of these

8.If $a_n>1$ for all $n\in N$ then $\log_{a_2} a_1+\log_{a_3} a_2+\ldots\ldots+\log_{a_n} a_{n-1}+\log_{a_1} a_n$ has the minimum value

a)1　　b)2　　c) 0　　d) none of these

9.The number of terms common between the series 1+2+3+4+8+………….. to 10 terms and 1+4+7+10+…….. to 100 term is

a) 6　　b) 4　　c) 5　　d) none of these

10.If a,b,c and d are different positive real numbers in H.P., then

a) ab>cd　　b) ac>bd　　c) ad>bc　　d) none of these