Three Dimensional Geometry

Rectangular Coordinate System in Space

1. if the origin is shifted (1, 2, -3) without changing the direction of the axes then find the new coordinates of the point (0, 4, 5) with respect to new frame.
2. find the coordinates of the point which divides the line joining points (2, 3, 4) and (3, -4, 7) in the ratio 5:3 internally.
3. prove that the three points A (3, -2, 4), B (1, 1, 1) and C (-1, 4, -2) are collinear.

Direction Ratios

4. find the direction ratios and direction cosines of the line joining the points A (6, -7, -1) and B (2, -3, 1).

Parallel Lines

5. if l_1, m_1, n_1 and l_2, m_2, n_2 are the d.c's of two intersecting lines, shows that the d.c's of two lines bisecting the angles between them are proportional to $l_1 \pm l_2$, $m_1 \pm n_2$.
6. Find the direction cosines of two lines which are connected by the relations l -5m + 3n = 0 and $7l^2 + 5m^2 - 3n^2 = 0$.

Angles between two Lines

7. if points P, Q are (2, 3, 4), (1, -2, 1), then prove that OP is perpendicular to OQ where O is (0, 0, 0)
8. find the perpendicular distance of point P (0, -1, 3) from the straight line passing through A (1, -3, 2) and B (2, -1, 4).

Problem

1. if a variable line in two adjacent positions has direction cosines l, m, n and $l + \delta l$, $m + \delta m$, $n + \delta n$, show that the small angle $\delta\theta$ between the two positions is given by $(\delta\theta)^2 = (\delta m)^2 + (\delta n)^2$.

Objective

1. the ratio, in which yz-plane divides the line joining (2, 4, 5) and (3, 4, 5), is

    (A) -2 : 3 (B) 2 : 3 (C) 3 : 2 (D) -3 : 2

2. A line makes angles α, β, γ, δ with the four diagonals of a cube. Then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta$ is equal to

    (A) 1 (B) 4/3 (C) 3/4 (D) 4/5

Exercise - 1

1. If a straight line makes angles α, β, γ, with the x, y, z axes respectively, then show that $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$.
2. show that A (-1, -3, 4), B (7, -4, 7) and D (1, -6, 10 from a rhombus.
3. Find the direction cosines l, m, n of two lines which are connected by the relations l + m + n = 0 and mn – 2nl – 2lm = 0.
4. Lines OA, OB are drawn from O with direction cosines proportional to 1, -2, -1; 3, -2, 3. Find the direction cosines of the normal to the plane AOB.
5. find the area of the triangle whose vertices are A (1, 2, 3), B (2, -1, 1) and C (1, 2,-4).
6. find the direction cosines of the line which is perpendicular to the lines with direction cosines proportional to (1, -2, -2), (0, 2, 1).
7. Show that the points (4, 7, 8), (2, 3, 4), (-1, -2, 1) and (1, 2, 5) are the vertices of a parallelogram.
8. if the vertices P, Q and R of a triangle have coordinates (2, 3, 5), (-1, 3, 2) and (3, 5, -2) respectively, then find the angles of the triangle PQR.
9. A (3, 2, 0), B (5, 3, 2) and (-9, 6, -3) are the vertices of a triangle ABC if bisector of $\angle BAC$meets BC at D, then co-ordinate of D are.

(A) $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$ (B) $\left(-\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$

(C) $\left(\frac{19}{8}, -\frac{57}{16}, \frac{17}{16}\right)$ (D) none of these

10. The angle between two diagonals of a cube is

(A) 30^{o} (B) 45^{o} (C) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (D) $\cos^{-1}\left(\frac{1}{3}\right)$

The Plane

1. Find the equation to the plane passing through the point (2, -1, 3) which is the foot of the perpendicular drawn from the origin to the plane.
2. Find the angles between the planes 2x – y + z = 11 and x + y + 2z = 3.
3. Find the equation of the plane passing through the points (2, 3, -4), (1, -1, 3) and parallel to the x-axis.
4. Find the equation of the plane through the line of intersection of the planes ax + by + cz + d = and αx+ βy + γz + δ = 0 and perpendicular to the xy- plane.
5. Show that the origin lies in the acute angel between the planes x + 2y + 2z – 9 = 0 and 4x – 3y + 12z + 13 = 0. Find the plane bisecting the angle between them and distinguish the acute angle bisector.

Problems

1. A variable plane passes through a fixed point $(\alpha, \ \beta, \ \gamma)$and meets the axes in A, B, C. Show that the locus of the point of intersection of the planes through A, B and C parallel to the co-ordinate planes is $\alpha x^{-1} + \beta y^{-1} + \gamma z^{-1} = 1$.
2. Find the image of the point (1, 3, 4) in the plane 2x – y + z + 3 = 0.

Objective

1. A plane passing through the point (-2, -2, 2) and containing the line joining the points (1, 1, 1) and (1, -1, 2) makes intercepts on the co-ordinate axes, the sum of whose length is

(A) 3 (B) 4 (C) 6 (D) 12

2. The points (0, -1, -1), (-4, 4, 4), (4, 5, 1) and (3, 9, 4) are

(A) collinear (B) coplanar (C) forming a square (D) none of these

3. consider the following statements:

Assertion (A): the plane y + z + 1 = 0 is parallel to the x-axis.

Reason (R): normal to the plane is parallel to the x-axis.

Of these statements:

(A) both A and R are true and R is the correct explanation of A

(B) both A and R are true and R is not a correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

4. the equation of the plane bisecting the acute angle between the planes 2x – y + 2z + 3 = 0 and 3x – 2y + 6z + 8 = 0 is

(A) 23x – 13y + 32z + 45 = 0 (B) 5x – y – 4z = 3

(C) 5x – y – 4z + 45 = 0 (C) 23x – 13y + 32z + 3 = 0

Exercise – 2

1. Find the equation of the plane trough (1, 0, -2) and perpendicular to each of the planes 2x + y – z – 2 = 0 and x – y – z - 3 = 0.
2. Find the plane passing through the line of intersection of the planes 2x + y – 3z + 5 = 0, x – 3y + z + 2 = 0 and the point (1, -2, 3).
3. Find the equation of the bisectors of the angles between the planes 2x – y – 2z – 6 = 0 and 3x + 2y – 6z – 12 = 0 and distinguish them.
4. A plane meets the co-ordinate axes in A, B, C such that the centroid of triangle ABC is (a, b, c). Then find the equation of the plane.

5. Find the equation of the plane parallel to the plane 4x – 3y + 2z + 1 = 0 and passing through the point (5, 1, -6).
6. find the equation of the right bisector plane of the segment joining (2, 3, 4) and (6, 7, 8).

7. the equation of the plane containing the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z}{2}$ and parallel to the line $\frac{x-3}{2} = \frac{y}{5} = \frac{z-2}{4}$ is

(A) x- 2y – z = 1 (B) x + 2y – 2z = 1 (C) x – 2y + 2z = 1 (D) x + 2y + 2z = 1

8. if a plane is passed through to middle point of the segment A (-2, 5, 1), (6, 1, 5) and is perpendicular to this line, then its equation is

(A) 2x – y + z = 4 (B) 2x + y + 2z = 4 (C) x – y + z = 5 (D) none of these

The Straight Line

1. Find the coordinates of the point where the line joining the points (2, -3, 1) and (3, -4 , -5)cuts the plane 2x + y + z = 7.
2. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane 3x + 4y – 6z + 1 = 0.
3. Find in symmetrical form, the equation of the line 3x + 2y – z – 4 = 0 = 4x + y – 2z + 3.
4. Find the equation of the plane passing through the intersection of planes 2x – 4y + 3z + 5 = 0, x + y + z = 6 and parallel to the straight line having direction ratios (1, -1, -1).

5. Find the equation of the plane which contains the two parallel lines $\frac{x+1}{3} = \frac{y-2}{2} = \frac{z}{1}$ and $\frac{x-3}{3} = \frac{y-2}{2} = \frac{z-1}{1}$.

6. A line with direction cosines proportional to (2, 7, -5) is drawn to intersect the lines $\frac{x-5}{23} = \frac{y-7}{-1} = \frac{z+2}{1}$ and $\frac{x+3}{-3} = \frac{y-3}{2} = \frac{z-6}{4}$. Find the co-ordinates of the points of intersection and the length intercepted on it. Also find the equation of intersecting straight line.

Shortest Distance Between Two Non Intersecting Lines

1. Find the shortest distance between the lines $\frac{x-1}{3} = \frac{y-8}{-1} = \frac{z-3}{1}, \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$. Also find the equation of the line of shortest distance.

Problem

1. Find the distance of the point (1, -2, 3) from the plane x – y – z = 5 measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$.

Objective

1. If the straight line $\frac{x-3}{-4} = \frac{y-4}{-7} = \frac{z+3}{13}$ lies in the planes 5x – y + z = a, then a is equal to

(A) 2 (B) -3 (C) 8 (D) 9

2. if the angle θ between the line $\frac{x+1}{1} = \frac{y-1}{2} = \frac{z-2}{2}$ and the plane 2x – y + $\sqrt{\lambda}$ z + 4 = 0 is such that $\sin\theta = \frac{1}{3}$, then the value of λ is

(A)3/4 (B) -4/3 (C) -3/5 (D) 5/3

3. A line with direction cosines proportional to 2, 1, 2 meets each of lines x = y + a = z and x + a = 2y = 2z, then the coordinates of each of the points of intersection are given by

(A) (3a, 2a, 3a), (a, a, 2a)
(B) (3a, 2a, 3a), (a, a, a)
(C) (3a, 3a, 3a), (a, a, a)
(D) (2a, 3a, 3a), (2a, a, a)

Excercise -3

1. Find the distance of the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane x + y + z = 17 from the point (3, 4, 5).
2. find the equations of the perpendicular from the point (1, 6, 3) to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{2}$. Find also the coordinates of the foot of perpendicular.
3. show that the length of shortest distance between the lines $\frac{x-2}{2} = \frac{y+1}{3} = \frac{z}{4}$;

   2x + 3y – 5z – 6 = 0 = 3x – 2y – z +3 is $\frac{97}{13\sqrt{6}}$

4. Find the length of the perpendicular from (1, -1, 2) to the plane 3x + 5y – 4z = 5 and also the coordinates of the foot of the perpendicular.
5. find the locus of the co-ordinate of points at a distance of $3\sqrt{11}$ units from (1, -1, 2) on a line passing through (1, -1, 2) and (3, 1, 1) and also find the coordinates of the point at the same distance from (1, -1, 2) on the given line.
6. find the equation of the plane containing (0, 7, -7) and $\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$

7. Prove that the angle between the line $\frac{x-1}{-1} = \frac{y+1}{1} = \frac{z-1}{1}$ and plane 3x + 2y – z = 4 is $\sin^{-1}\left(\frac{-2}{\sqrt{42}}\right)$.

8. equation of straight line which passes through the point P (1, 0, -3) and Q (-2, 1, -4) is

(A) $\frac{x-2}{-3} = \frac{y+1}{1} = \frac{z-4}{-1}$
(B) $\frac{x-1}{3} = \frac{y}{1} = \frac{z+3}{1}$
(C) $\frac{x-1/2}{-3} = \frac{y-1}{1} = \frac{z+4}{-1}$
(D) $\frac{x-1}{-3} = \frac{y}{1} = \frac{z+3}{1}$

9. The distance of the point (-2, 3, -4) from the line $\frac{x+1}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane, 4x + 12y – 3z + 1 = 0 is

(A) $\frac{17}{3}$ (B) $\frac{17}{2}$ (C) $\frac{13}{3}$ (D) $\frac{13}{2}$

Miscellaneous Solved Problems

1. find the equation of the plane containing the line $\frac{y}{6} + \frac{z}{c} = 1$, x = 0 and parallel to the line $\frac{y}{6} - \frac{z}{c} = 1$, y = 0
2. the sum of the perpendicular distances taken with proper sign of any number of fixed points from a variable plane is zero. Show htat the variable plane passes through a fixed point.

3. find the points on the line $\frac{x-6}{3}$ = -(y – 7) = (z - 4) and $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$ which are nearest to each other. Hence find the shortest distance between the given lines.
4.Show that the reflection of the plane a'x + b'y + c'z + d' = 0 in the plane ax + by + cz + d = 0 is the plane

   2(aa' + bb' + cc') (ax + by + cz + d) = 0 ($a^2 + b^2 + c^2$) (a'x + b'y + c'z + d').
5. find the equation of the plane through the point $(\alpha', \beta', \gamma')$ and the line $\frac{x-\alpha}{1} = \frac{y+\beta}{m} = \frac{z-\gamma}{n}$.
6. what do you understand by the projection of a line on a given plane? Find the equations of the projection of the line $\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-3}{4}$ on the plane x + 2y + z = 6

Objective

1. A plane which passes through the point (3, 2, 0) and containing the line $\frac{x-4}{1} = \frac{y-7}{5} = \frac{z-4}{4}$, may be

(A) x – y + z -1 = 0 (B) x – y + z – 5 = 0 (C) x – 2y – z – 1 = 0 (D) 2x – y + z -5 = 0

2. A plane mirror is placed at the origin so that the direction ratios of its normal are 1, -1, 1. A ray of light, coming along the positive direction of the x-axis, strikes the mirror. The direction cosines of the reflected ray are

(A) $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ (B) $-\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$ (C) $-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}$ (D) $-\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

3. The equation of the plane containing the line $\frac{x-\alpha}{1} = \frac{y+\beta}{m} = \frac{z-\gamma}{n}$ is a$(x - \alpha)$ + b$(y + \beta)$ + c$(z - \gamma)$ = 0, where al + bm + cn is equal to

(A) 1 (B) -1 (C) 2 (D) 0

4. the shortest distance between the two straight lines $\frac{x-4/3}{2} = \frac{y+6/5}{3} = \frac{z-3/2}{4}$ and $\frac{5y+6}{8} = \frac{2z-3}{9} = \frac{3x-4}{5}$ is

(A) $\sqrt{29}$ (B) 3 (C) 0 (D) $6\sqrt{10}$

5. A straight line passes through the point (2, -1, -1). It is parallel to the plane 4x + y + z +2 = 0 and is perpendicular to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z-5}{1}$. the equation of the straight line are

(A) $\frac{x-2}{4} = \frac{y+1}{1} = \frac{z+1}{1}$ (B) $\frac{x+2}{4} = \frac{y-1}{1} = \frac{z-1}{1}$

(C) $\frac{x-2}{-1} = \frac{y+1}{1} = \frac{z+1}{3}$ (D) $\frac{x+2}{-1} = \frac{y-1}{1} = \frac{z-1}{1}$

6. the equations of the straight line through the origin and parallel to the line (b + c)x + (c + a)y + (a + b)z = k = (b – c)x + (c – a)y + (a – b)z are

(A) $\frac{x}{b^2-c^2} = \frac{y}{c^2-a^2} = \frac{z}{a^2-b^2}$ (B) $\frac{x}{b} = \frac{y}{c} = \frac{z}{a}$

(C) $\frac{x}{a^2-bc} = \frac{y}{b^2-ca} = \frac{z}{c^2-ab}$ (D) none of these

Assignments

Section – I

Part - A

1. A line make the same angle θ with each of the x and z-axis. If the single β which it make with the y-axis, is such that $\sin^2\beta = 3\sin^2\theta$, then find the value of $\cos^2\theta$.
2. Find the area of the triangle whose vertices are (0, 0, 0), (5, 2, 6).
3. Prove that the straight lines whose direction cosines are given by the relations al + bm + cn = 0 and fmn + gnl + hlm = 0 are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$.
4. Prove that the theree lines from the origin O, with direction cosines l_1, m_1, n_1 : l_2, m_2, n_2 : l_3, m_3, n_3, are coplanar if $\begin{pmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{pmatrix} = 0$.

5. find the image of the line $\frac{x-1}{9} = \frac{y-2}{-1} = \frac{z+3}{-3}$ in the plane 3x -3y + 10z – 26 = 0.

6. find the equation of plane through the points (2, 2, 1) and (9, 3, 6) and perpendicular to plane 2x + 6y + 6z – 1 = 0.
7. Prove that the equation of a plane perpendicular to line x – 1 = 2 – y $= \frac{z+1}{2}$ and passing through (2,3,1) is x – y + 2z = 1.
8. find the equation of line passing through (-1, -2, -3) and perpendicular to plane 3x + 2y + 3z + 5 = 0 .
9. Find the equation of the plane through the line of intersection of planes ax + by + cz + d = 0 and a'x + b'y + c'z + d' = 0 and parallel to the line y = 0, z = 0.

10. The point P’ $(\alpha', \beta', \gamma')$ is the mirror image of point P (α, β, γ), in a line passing through the origin

with direction cosines l, m, n. Prove that $\frac{\alpha+\alpha'}{l} = \frac{\beta+\beta'}{m} = \frac{\gamma+\gamma'}{n} = 2\,(l\alpha + m\beta + n\gamma)$.

11. Find the equation of the plane which bisects the join of P(x_1, y_1, z_1) and (x_2, y_2, z_2) perpendicularly.

12. The plane lx + my = 0 is rotated about its line of intersection with the plane z = 0, through an angle α. Prove that the equation of the plane in its new position is

Lx + my $\pm$ z tan $\alpha\sqrt{(l^2 + m^2)}$ = 0.

Part – B

(Multi Choice Single Correct)

1. The projectors of a directed line segment on the co-ordinate axes are 12, 4, 3. Then direction cosines of the line are

(A) $\frac{12}{13}, -\frac{4}{13}, \frac{3}{3}$ (B) $-\frac{12}{13}, -\frac{4}{13}, \frac{3}{3}$

(C) $\frac{12}{13}, \frac{4}{13}, \frac{3}{3}$ (D) x + y – 8z – 8 = 0

2. OP = 3 with direction ratios -1, 2, -2, then coordinates of P if ‘O’ is, origin is

(A) (-1, 2, -2) (B) (1, 2, 2) (C) $\left(-\frac{1}{9}, \frac{2}{9}, -\frac{2}{9}\right)$ (D) (3, 6, -9)

3. the direction cosines of a line equally inclined to three mutually perpendicular lines having direction as l_1, m_1, n_1 : l_2, m_2, n_2 : l_3, m_3, n_3 are

(A) $l_1 + l_2 + l_3$, $m_1 + m_2 + m_3 + n_1 + n_2 + n_3$ (B) $\frac{l_1+l_2+l_3}{\sqrt{3}}, \frac{m_1+m_2+m_3}{\sqrt{3}}, \frac{n_1+n_2+n_3}{\sqrt{3}}$

(C) $\frac{l_1+l_2+l_3}{3}, \frac{m_1+m_2+m_3}{3}, \frac{n_1+n_2+n_3}{3}$ (D) none of these

4. if α, β, γ are the angles which a directed line makes with the positive directions of co-ordinates axes, then $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is equal to

(A) 1 (B) 2 (c) 3 (D) none of these

5. the distance of the points P (a,b,c) from x- axis is

(A) $\sqrt{b^2 + c^2}$ (B) $\sqrt{a^2 + c^2}$

(C) $\sqrt{a^2 + b^2}$ (C) none of these

6. A is a point (3, 7, 5) and B is the point (-3, 2, 6). The projection of AB on the line which joins the points (7, 9, 4) and (4, 5, -8) is

(A) 26 (B) 2 (C) 13 (D) 4

7. if P_1P_2 is perpendicular to P_2P_3, then the value of k, where P_1(k, 1, -1), P_2(2k, 0, 2) is 4, then its z-coordinates is

(A) 2 (B) 1 (C) -1 (D) -2

8.If the x – coordinates of a point P on the joint of Q (2,2,1) and R(5,1,-2) is 4, then its z – coordinates is

a) 2 b) 1 c) -1 d) -2

9. The coordinate of foot of perpendicular drawn from A (1, 0, 3) to the join of B (4, 7, 1) and C (3, 5, 3) are

(A) $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$ (B) (5, 7, 17)

(C) $\left(\frac{5}{7}, -\frac{7}{3}, \frac{17}{3}\right)$ (D) $\left(-\frac{5}{3}, \frac{7}{3}, -\frac{17}{3}\right)$

10. A variable line through the origin intersects the parallel planes x + y + 2z + 9 = 0 and x + y + 2z – 6 = 0 at P and Q. the ratio OP : OQ is

(A) 3 : 2 (B) 2 : 3 (C) 1 : 1 (D) 5 : 2

11. which one of the following is best condition for the plane ax + by + cz + d = 0 to intersect the x and the y-axes at equal angles

(A) … and …　　(B) a = -b　　(C) a = b　　(D) $a^2 + b^2 = 1$

12.the condition that line $\frac{x-x_1}{1} = \frac{y-y_1}{m} = \frac{z-z_1}{n}$ lies in the plane ax + by + cz + d = 0 is

(A) $ax_1 + by_1 + cz_1 + d = 0$ and $al + bm + cn \neq 0$
(B) $al + bm + cn = 0$ and $ax_1 + by_1 + cz_1 + d \neq 0$
(C) $ax_1 + by_1 + cz_1 + d = 0$ and $al + bm + cn = 0$
(D) $ax_1 + by_1 + cz_1 = 0$ and $al + bm + cn = 0$

13. the line $\frac{x-1}{1} = \frac{y-3}{2} = \frac{z-4}{3}$ parallel to the plane

(A) 2x + y + 2z + 3 = 0　　(B) 2x – y – 2z = 3
(C) 21x – 12y + z = 0　　(D) 2x + y – 2z = 0

14. the equation of the plane containing the line 2x – 5y + 2z = 6, 2x + 3y – z = 5 and parallel to the line $\frac{x}{1} = \frac{y}{-6} = \frac{z}{7}$ is

(A) 6x + y – 10 = 0　　(B) 6x + y – 16 = 0
(C) 12x + 2y – 1 = 0　　(D) 6x + y + 16 = 0

15. the lines $\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z}{3}$ and $\frac{9x-16}{13} = \frac{9y-1}{7} = \frac{z}{-1}$ are

(A) coplanar　　(B) parallel　　(C) skew　　(D) none of these

Multi Choice Multi Correct

1. the equation of line passing through (2, 3, -1) and lying in the plane 2x + y – 5z = 12 and perpendicular to the line 2x + y -5z -12 = 0 = 4x – 3y + 7z is

(A) $\frac{x-2}{3} = \frac{y-3}{-1} = \frac{z+1}{1}$　　(B) $\frac{x-1}{-3} = \frac{y-5}{1} = \frac{z+1}{-1}$
(C) $\frac{x-5}{3} = \frac{y-2}{-1} = \frac{z}{1}$　　(D) $\frac{x-2}{-3} = \frac{y-2}{1} = \frac{z+1}{7}$

2. Let A (1, -1, 1) and B (-1, 1, -1) be the vertices of triangle ABC such that $\angle A = \angle B$. The locus of the vertex C is a plane which will be perpendicular to

(A) x + 2y + z = 0　　(B) 2x + y – z = 0　　(C) x – y + z = 0　　(D) x – 2y – 3z = 5

3. if lines x = y = z, x = $\frac{y}{2} = \frac{z}{3}$ and the third line passing through (1, 1, 1) from a triangle of area $\sqrt{6}$ units, then point of intersection of third line with second line will lie on

(A) $\frac{x-2}{3} = \frac{y-4}{8} = \frac{z-6}{9}$　　(B) x + 2y + z = 16
(C) x – 2y – z = 16　　(D) $\frac{x-3}{3} = \frac{y-4}{8} = \frac{z-3}{5}$

Numerical Based Type

1. shortest distance between lines $\frac{x-6}{1} = \frac{y-2}{-2} = \frac{z-2}{2}$ and $\frac{x+4}{3} = \frac{y}{-2} = \frac{z+1}{-2}$ is

2. the square of the projection of the line segment joining the points (1, 2, 3) and (4, 5, 6) on the plane 2x + y + z = 1 is

Linked Comprehension Type

Read the following write up carefully and answer the following questions:

A line L_1 passing through a point (-4, -6, 1) and is parallel to a line $\frac{x-1}{3} = \frac{y+2}{5} = \frac{z-2}{-2}$ and second Line L_2 is the line of intersection of planes 3x – 2y+z+5 =0 and 2x+3y+4z – 4=0. Both lines L_1 and L_2 are coplanar.

1. the point of intersection of lines L_1 and L_2 is

   (A) (2, 4, 3) (B) (-2, 4, -3) (C) (2, 4, -3) (D) none of these

2. The equation of plane containing line L_1 and passing through (1, -2, 2)

   (A) x – y – z = 1 (B) 2x + y – z = 3 (C) (x – y + 2z = 3) (D) none of these

3. Direction ratio of the line L_2 is

   (A) (10, 11, -13) (B)(11, -10, 13) (C) (11, 10, -13) (D) none to these

4. Distance between line L_1 and $\frac{x-1}{3} = \frac{y+2}{5} = \frac{z-2}{-2}$ is

   (A) $13\sqrt{\frac{3}{25}}$ (B) $13\sqrt{\frac{3}{38}}$ (C) $12\sqrt{\frac{3}{38}}$ (D) none of these

5. Angle between planes 3x – 2y + 5 = 0 and 2x + 3y + 4z – 4 = 0 is

   (A) $\cos^{-1}\frac{2}{\sqrt{203}}$ (B) $\cos^{-1}\frac{4}{\sqrt{406}}$ (C) $\cos^{-1}\frac{4}{\sqrt{203}}$ (D) none of these

Matrix - Match Type

Each question contains statements given in two columns which have to be matched. Statements (A,B,C,D) in column 1 have to be matched with statements (p,q,r,s) in column II.

1. consider the following linear equations

   ax + by + cz = 0
   bx + cy + az = 0
   cx + ay + bz = 0

Colum 1	Colum 2
(A) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 = ab + bc + ca$	(p) the equations represent planes meeting only at a single point.
(B) $a + b + c = 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(q)the equation represent the line x = y = z.
(C) $a + b + c \neq 0$ and $a^2 + b^2 + c^2 \neq ab + bc + ca$	(r) the equations represent identical planes
(D) $a + b + c = 0$ and $a^2 + b^2 + c^2 = ab + bc + c$	(s) the equations represent the whole of the three dimensional space.

Section - II

1. The general equation of the plane which is parallel to the x-axis is

   (A) by + cz + d = 0, $b \neq 0$, $c \neq 0$
   (B) ax + by + cz + d = 0, $a \neq 0$, $b \neq 0$, $c \neq 0$
   (C) ax + cz + d = 0, $a \neq 0$, $c \neq 0$
   (D) ax + by + d = 0, $a \neq$, $b \neq 0$

2. The locus of a point which is equidistant from the x- axis and the y- axis is

   (A) the plane x = y (B) the plane x = -y
   (C) the pair of planes $x = \pm y$ (D) none of these

3. if (p_1. q_1, r_1) be the image of (p, q, r) in the plane ax + by + cz + d = 0, then

   (A) $\frac{p_1 - p}{a} = \frac{q_1 - q}{b} = \frac{r_1 - r}{c}$ (B) $a(p + p_1) + (q + q_1) + c(r + r_1) + 2d = 0$
   (C) both (A) and (B) (D) none of these

4. The mirror images of the point (3, 4, 5) in the coordinate planes are A, B and C. The centroid of the triangle ABC is

(a) (0, 0, 0) (B) (-3, -4, -5) (C) $\left(\frac{3}{3}, \frac{4}{3}, \frac{5}{3}\right)$ (D) (1, 1, 1)

5. which of the following does not represent a straight line

(A) ax + by + cz + d = 0, a'x + by + cz + d = 0 (a $\neq$ a')

(B) ax + by + cz + d = 0, ax + b'y + cz + d = 0 (b $\neq$ b')

(C) ax + by + cz + d = 0, ax + by + c'z + d = 0 (c $\neq$ c')

(D) ax + by + cz + d = 0, ax + by + cz + d' = 0 (d $\neq$ d)

6. A plane makes intercepts OA, OB, OC whose measurements are a, b, c on the axes OX, OY, OZ. The area of the ΔABC is

(A) $\frac{1}{2}$ (ab + bc + c a) (B) $\frac{1}{2}$ $(a^2b^2 + b^2c^2 + c^2a^2)^{1/2}$

(C) $\frac{1}{2}$ abc (a + b + c) (D) $\frac{1}{2}$ (a + b + c) (ab + bc + c a)

7. The ratio of the distances from the points (1, -1, 3) and (3, 3, 3) to the plans 5x + 2y – 7z + 9 = 0 is

(A) 2 : 1 (B) 1 : 3 (C) 1 : 1 (D) 3 : 2

8. A perpendicular is drawn from a point (1, 6, 3) to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Coordinates of the foot of perpendicular are

(A) (1, 3, 5) (B) (0, 3, -2) (C) (2, 4, -5) (D) (1, 3, 4)

9. the straight lines, whose direction cosines are l_i , m_i , n_i which are the roots of al + bm + cn = 0, $fl^2 + gm^2 + hn^2 = 0$, are parallel if

(A) $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$ (B) $\frac{a}{f} + \frac{b}{g} + \frac{c}{h} = 0$ (C) $\frac{f}{a^2} + \frac{g}{b^2} + \frac{h}{c^2} = 0$

(D) $\frac{a^2}{f} + \frac{b^2}{g} + \frac{c^2}{h} = 0$

126. the distance of the point (-2, -5, 7) from the point of intersection of the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and the planes 2x + y – z = 2 is

(A) $4\sqrt{29}$ (B) $\sqrt{78}$ (C) $2\sqrt{29}$ (D) $2\sqrt{78}$