



THE GURUKUL INSTITUTE

PLOT 5C, 2ND FLOOR, GANAPATI COMPLEX, SEC-13, OPP. JAIPURIA
SCHOOL, VASUNDHARA, GHAZIABAD (U.P)

MATHS TEST

CLASS XII

1. If $y = \sqrt{x^2 + 1} - \log\left(\frac{1}{x} + \sqrt{1 + \frac{1}{x^2}}\right)$, find $\frac{dy}{dx}$.
2. Differentiate, $x^x \sin^{-1}\sqrt{x}$ w.r.t x .
3. Verify Rolle's Theorem for the following function: $F(x) = x^{2/3}$ on $[-1, 1]$.
4. A balloon which always remains spherical is being inflated by pumping in gas at the rate of $900\text{cm}^3/\text{sec}$. Find the rate at which the radius of the balloon is increasing when the radius of the balloon is 15 cm.
5. Show that the height of a cylinder of maximum volume that can be inscribed in a sphere of radius R is $2R/\sqrt{3}$.
6. Show that volume of greatest cylinder which can be inscribed in a cone of height h and semi vertical angle α is $\frac{4}{27}\pi h^3 \tan^2\alpha$.
7. Integrate : $\int \cos^4 x \, dx$.
8. Prove that : $\int_0^1 \sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = \pi/2 - \log 2$.
9. Sketch the region enclosed between the circle $x^2+y^2 = 1$ and $x^2+(y-1)^2 = 1$. Also find the area of the region using integration.
10. Solve the following differential equation: $(1+y^2)(1+\log x)dx + x \, dy = 0$, given that when $x=1, y=1$.
11. Solve : $\frac{dy}{dx} = \frac{x e^x \log x + e^x}{x \cos y}$.
12. $(1+x^2)\frac{dy}{dx} - 2xy = (x^2 + 2)(x^2 + 1)$.
13. Find the Cartesian and vector equations of a line which passes through the point $(-4, 2, -3)$ and is parallel to the line $\frac{-x-2}{4} = \frac{y+3}{-2} = \frac{2z-6}{3}$.
14. Find the coordinates of the foot of the perpendicular drawn from the point $A(1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$.
15. Find the point on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance $3\sqrt{2}$ from the point $(1, 2, 3)$.
16. A small firm manufactures items A and B. the total number of items A and B that it can manufacture in a day is at the most 24. Item A takes one hour to make while item B takes only half an hour. The maximum time available per day is 16 hours. If the profit on one unit of item A be Rs. 300 and one unit of item B be Rs. 160, how many of each type of item be produced to maximize the profit? Solve the problem graphically.
17. For what value of k , is the following function continuous at $x = 0$:

$$f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

18. Using the properties of determinants, prove the following :

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3.$$