THE GURUKUL INSTITUTE

PLOT 5C, 2ND FLOOR, GANAPATI COMPLEX, SEC-13, OPP. JAIPURIA SCHOOL, VASUNDHARA,GHAZIABAD (U.P)

TEST CLASS - XIIth

1. Find the value of x and y if: $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$
2. Using properties of determinants prove that:
$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)\,(b-c)\,(c-a)\,(a+b+c)$$
3. If A = $\begin{bmatrix} -1 & -1 \\ 2 & -2 \end{bmatrix}$ show that $A^2 + 3A + 4I = 0$. Hence find A^{-1}
4. Evaluate : $\tan^{-1} 3/4 + \tan^{-1} 3/5 - \tan^{-1} 8/9$
5. Find the value of k for which the function f(x) = $\begin{cases} kx+5, & x \le 2 \\ x-1, & x > 2 \end{cases}$ is continuous at x=2.
6. Differenciate : $\tan^{-1}\left[\dfrac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ w. r. t. $\cos^{-1}x^2$
7. Using the properties of determinants prove that :
$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$$
8. If f(x) $= \dfrac{4x+3}{6x-4}$, $x \neq 2/3$, show that 'f0f' is an identity function for all $x \neq 2/3$, what is the inverse of 'f' ?
9. Prove that, $\sin^{-1} 12/13 + \cos^{-1} 4/5 + \tan^{-1} 63/16 = \pi$.
10. Let R → R be defined by f(x) = 2x – 3 and g: R→R be defined by g(x) = $\dfrac{x+3}{2}$ show that $fog = I_R = gof$.
11. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that :
$$\frac{dy}{dx} = \frac{\sqrt{(1-y^2)}}{\sqrt{(1-x^2)}}$$
12. If $x^p \cdot y^q = (x+y)^{p+q}$, show that $dy/dx = y/x$
13. Solve the following system of equations by matrix method:
3x-4y+ 2z = -1 ; 2x+ 3y+ 5z = 7 ; x+z =2
14. A wire of length 25 m is to be cut into two pieces, one of the pieces is to be made into a square and the other into a circle. What should be the length of each piece to that the combined area of the square and the circle is to be minimum ?
15. Show that the height of the right circular cylinder of maximum volume that can be inscribed in a sphere of radius R is $2R/\sqrt{3}$
16. A window is the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window so as to admit maximum light through it.