MATHEMATICS

Trigonometric Ratios & Identities

Basic Questions

1.Express $\tan\theta$ in terms of $\cos\theta$.

2.Prove that $2(\sin^6\theta+\cos^6\theta) - 3(\sin^4\theta+\cos^4\theta)+1=0$.

3.Prove that $(\sin\theta+\operatorname{cosec}\theta)^2+(\cos\theta+\sec^2\theta) \geq 9$.

4.If $\tan^2\theta= 1 - e^2$ then show that $\sec\theta+ \tan^3\theta \operatorname{cosec}\theta=(2 - e^2)^{3/2}$.

Trigonometric Ratios of any Angle

5.Find the general value of θ satisfying both $\sin\theta = -1/2$ and $\tan\theta =1/\sqrt{3}$.

6.(i)Find the value of $\tan1^0 \tan2^0\tan3^0\ldots\ldots\ldots\ldots\ldots\tan89^0$.

(ii)Prove that $\sin^25^0+\sin^210^0+\ldots\ldots\ldots\ldots+\sin^290^0 = 19/2$.

Trigonometric Ratios of Compound Angles

7.Prove that $\tan 70^0= 2\tan50^0 +\tan20^0$.

8.If $A+B=45^0$, show that$(1+\tan A)(1+\tan B) =2$.

9.(i)Show that $\cot(\pi/4+x) \cot(\pi/4 - x)=1$

ii) Give n 3 $\tan\alpha\tan\beta =1$, show that $2\cos(\alpha+\beta) = \cos(\alpha - \beta)$

iii)Eliminate θ if $\tan(\theta - \alpha)=a$ and $\tan(\theta+\alpha)=b$.

Trigonometry Ratios of Multiples of an Angle.

10.Find the values of (i) $\sin 18^0$ (ii)$\tan 15^0$.

11. Find the values of $\sin 67\ 1^0_{\overline{2}}$

12.(i)Prove that $2\sin^2\theta +4\cos(\theta+\alpha)\sin\alpha.\sin\theta + \cos2(\theta+\alpha)$ is independent of θ.

(ii)If $\tan\theta = b/a$, then find the value of $a\cos2\theta +b\sin2\theta$.

13.If α,β and γ are in A.P., show that $\cot\beta = \dfrac{\sin\alpha - \sin\gamma}{\cos\gamma - \cos\alpha}$

14.Show that $\sin 12^0.\sin48^0.\sin54^0 = 1/8$.

15.If $\sin x+ \sin y = \sqrt{3}(\cos y - \cos x)$ show that $\sin3x+\sin3y=0$.

16.If $\dfrac{\cos(A - B)}{\cos(A+B)} + \dfrac{\cos(C+D)}{\cos(C-D)} = 0$, then prove that $\tan A.\tan B.\tan C.\tan D = -1$.

17.If $\dfrac{\sin(\theta+\alpha)}{\cos(\theta - \alpha)} = \dfrac{1 - m}{1+m}$, prove that $\tan(\pi/4 - \theta)\tan(\pi/4 - \alpha) =m$.

18.Prove that $(\cos\alpha - \cos\beta)^2 +(\sin\alpha - \sin\beta)^2 = 4\sin^2\left(\dfrac{\alpha - \beta}{2}\right)$

19.Prove that $\sin\alpha+\sin(\alpha+2\pi/3) + \sin(\alpha+4\pi/3) =0$.

20.Find the value of $\cos 22\ 1^0_{\overline{2}}$

IDENTITIES

1.If $A+B+C =\pi$, prove that $\dfrac{\sin 2A+\sin2B+\sin2C}{\cos A+\cos B+\cos C-1} = 8\cos A/2 \cos B/2 \cos C/2$.

2.If A,B and C are the angles of a triangle, show that $\tan^2 A/2 + \tan^2B/2+\tan^2C/2 \geq 1$.

3.Find the maximum and minimum values of $a\cos\theta+ b\sin\theta$.

4.(i)If $A+B+C =\pi$, then prove that

$\sin(B+2C) + \sin(C+2A)+\sin(A+2B) = 4\sin\dfrac{(B - C)}{2} . \sin\dfrac{(C - A)}{2}. \sin\dfrac{(A - B)}{2}$.

(ii)If $A+B+C =\pi$, then find the minimum value of $\cot^2A + \cot^2B+\cot^2C$.

SOME USEFUL FORMULE

1.Simplify the product $\cos A \cos 2A . \cos 2^2 A \ldots\ldots\ldots\ldots \cos 2^{n-1} A$.

2.Prove that $\cos\pi/65 \cos 2\pi/65 \cos 4\pi/65 \cos 8\pi/65 \cos 16\pi/65 \cos 32\pi/65 = 1/64$.

3.Prove that $\cos 6^0 . \cos 42^0 . \cos 66^0 . \cos 78^0 = 1/16$.

PROBLEMS

1.Let $0<A,B<\pi/2$ satisfy the equations $3\sin^2 A + 2\sin^2 B = 1$ and $3\sin 2A - 2\sin 2B = 0$. Prove that $A + 2B = \pi/2$.

2.Prove that $\cos 2\pi/15 . \cos 4\pi/15 \cos 8\pi/15 . \cos 16\pi/15 = 1/16$.

3.If $\cos A = m \cos B$, then prove that $\cot \frac{A+B}{2} = \frac{m+1}{m-1} \tan \frac{B-A}{2}$

4.If $a\cos 2\theta + b \sin 2\theta = c$ has α and β as its solutions, then prove that :

$\tan\alpha + \tan\beta = \frac{2b}{(c+a)}$, $\tan\alpha\tan\beta = \left(\frac{c-a}{c+a}\right)$

5.Prove that $5\cos\theta + 3\cos(\theta+\pi/3) + 3$ lies between -4 and 10.

6.(i)For all θ in $[0,\pi/2]$, show that $\cos(\sin\theta) > \sin(\cos\theta)$.

ii) Find the smallest positive number p for which the equation $\cos(p \sin x) = \sin(p \cos x)$ has a solution $x \in [0,2\pi]$.

7.If ABC is a triangle and tan A/2, tan B/2, tanC/2 are in H.P., then find the minimum value of cot B/2.

8.Let $\cos A + \cos B + \cos C = 3/2$ in a triangle ABC. Show that the triangle is equilateral.

9.Prove that $1 + \cot\theta \leq \cot\theta/2$ for $0<\theta<\pi$. Find θ when equality sign holds.

OBJECTIVE

Multiple Choice (Single Correct)

1.If $3\sin\theta + 5\cos\theta = 5$, then the value of $5\sin\theta - 3\cos\theta$ is equal to

a) 5 b) 3 c) 4 d) none of these

2.If $\tan\theta = \sqrt{n}$ for some non – square natural number n, then sec 2θ is

a) a rational number b) an irrational number

c) a positive number d) none of these

3.The minimum value of $\cos(\cos x)$ is

a) 0 b) $-\cos 1$ c) $\cos 1$ d) -1

4.If $\sin x = \cos^2 x$, then $\cos^2 x(1+\cos^2 x)$ is equal to

a) 0 b) 1 c) 2 d) none of these

5.The maximum value of $4\sin^2 x + 3\cos^2 x + \sin x/2 + \cos x/2$ is.

a) $4+\sqrt{2}$ b) $3+\sqrt{2}$ c) 9 d) 4.

6.If α and β are the solutions of $\sin^2 x + a \sin x + b = 0$ as well as that of $\cos^2 x + c \cos x + d = 0$, then $\sin(\alpha+\beta)$ is equal to.

a) $\frac{2bd}{b^2+d^2}$ b) $\frac{a^2+c^2}{2ac}$ c) $\frac{b^2+d^2}{2bd}$ d) $\frac{2ac}{a^2+c^2}$

7.If $\sin\alpha$, $\sin\beta$ and $\cos\alpha$ are in G.P, then roots of the equation $x^2 + 2x\cot\beta + 1 = 0$ are always

a) equal b) real c) imaginary d) greater than 1

8.If in a triangle ABC, $\sin^2 A + \sin^2 B + \sin^2 C = 2$, then the triangle is always

a) isosceles triangle b) right angled c) acute angled

d) obtuse angled

9.If $\theta \neq (2n+1)\pi/2$, n$\in$l then the least value of $(\sin\theta+\operatorname{cosec}\theta)^2 + (\cos\theta+\sec\theta)^2$ is

a) 2 b) 4 c) 8 d) 9.

10.If $4n\alpha = \pi$, then $\cot\alpha \cot 2\alpha \cot 3\alpha \ldots\ldots\ldots \cot(2n-1)\alpha$ is equal to

a) 0 b) 1 c) n d) none of these

11.If $\cos 5\theta = a\cos\theta + b\cos^3\theta + c\cos^5\theta + d$, then

a) a = 20 b) b = -30 c)a+b+c=2 d) a+b+c+d =1

12.If in a triangle ABC $(\sin A+\sin B+\sin C)(\sin A+\sin B-\sin C)=3\sin A\sin B$, then angle C is equal to
a) 30^0 b) 45^0 c) 60^0 d) 75^0

ASSIGNMENTS

Section – I

Part – A

Level – I

1.Prove the following:
i) $\cos^2\theta+\cos^2(\theta+120^0)+\cos^2(\theta-120^0)=3/2$.
ii)$(\sec A-\operatorname{cosec}A)(1+\tan A+\cot A)=\tan A\sec A-\cot A\operatorname{cosec}A$.
iii)$(\operatorname{cosec}\theta-\sec\theta)(\cot\theta-\tan\theta)=(\operatorname{cosec}\theta+\sec\theta)(\sec\theta\operatorname{cosec}\theta-2)$.
2.In a ΔABC, prove that $(\sin A+\sin B)(\sin B+\sin C)(\sin C+\sin A)>\sin A\sin B\sin C$.
3.If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi/2$ and $(x-y)^2+(y-z)^2+(z-x)^2=0$,
Then prove that $x^2+y^2+z^2=1$.
4.Prove that $2\cos x-\cos 3x-\cos 5x=16\cos^3x\sin^2x$.
5.Find the value of $\sin^4\pi/8+\sin^4 3\pi/8+\sin^4 5\pi/8+\sin^4 7\pi/8$.